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Debt crises, fast and slow*

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Abstract

We build a dynamic model to study how shifts in investors' beliefs can drive either slow-moving debt crises or rollover crises. We show that the threat of slow-moving crises does not necessarily motivate deleveraging: in a recession, unless debt is close enough to the threshold at which the economy becomes vulnerable to such crises, optimizing governments keep borrowing, gambling on economic recovery. The incentive to deleverage is instead strong when the economy is vulnerable to rollover crises at low levels of debt. We show that equilibrium multiplicity remains pervasive independently of bond maturity. In general, short maturities induce more deleveraging.

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1 Introduction

After the global financial crisis, the average public debt to GDP ratio in advanced countries rose from below 80 percent to well above 100 percent at the end of 2008. After 2020, the global distress of the COVID-19 pandemic is sparking a further hike in this ratio, expected to end up substantially higher—and raise a host of issues in financial and macroeconomic stability. The academic and policy literature has long reflected on the possibility that countries with relatively high debt face disruptive belief-driven turmoil in the sovereign bond market, increasing borrowing costs that feed unsustainable debt dynamics, or even resulting in sudden stops and rollover crises. The exercise pursued by this literature is far from a theoretical *curiosum*. The turmoil in the euro area after 2010 provides a vivid and striking example of the widespread disruption caused by this type of crises even among advanced countries.¹

In this paper, we reconsider the logic of debt crises, specifying a stylised model suitable to address two key open issues in the literature. First, sovereign risk (slow-moving) and rollover (fast) crises appear to be pervasive in the data: under what conditions sovereigns may face hikes in borrowing costs, or lose market access, due to market beliefs coordinating on a “bad equilibrium”? In particular, is lengthening the debt maturity an effective way to shield countries from these adverse scenarios? Second, and most crucially, is the threat of belief-driven crises enough to motivate optimal deleveraging—as opposed to borrowing more, gambling on a future economic recovery?

We address these questions by specifying a model with the same setting of [Conesa and Kehoe \(2017\)](#), except that the timing of the auction is that of [Calvo \(1988\)](#)²—hence the model does not feature the rollover crises around the bond auction envisioned by [Cole and Kehoe \(2000\)](#). Using this standard framework, we reconsider the mechanisms through which market beliefs may cause a variety of sovereign crises, eliciting different policy responses by optimizing government.

As in [Calvo \(1988\)](#), in our model market expectations of future default determine the bond price today. Conditional on this price, a discretionary government optimally adjusts its fiscal surplus and debt repayment/issuance decisions. Market beliefs about future default thus impinge on the equilibrium fiscal policy and debt dynamics, as well as on the optimal debt tolerance thresholds based on which the government takes its default decisions. We allow

¹In the words of the ECB president Mario Draghi: “The assessment of the Governing Council is that we are in [...] a ‘bad equilibrium’, namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios.” ECB Press Conference, Transcript from the Q&A, September 6 2012.

²See the discussion in [Lorenzoni and Werning \(2019\)](#), [Corsetti and Dedola \(2016\)](#) and [Ayres et al. \(2018\)](#) in the Calvo tradition.

for three sets of beliefs: “optimistic”, “pessimistic” and “extreme”. Beliefs are “optimistic” if, when the equilibrium is not unique, investors always coordinate on the equilibrium with the highest bond price. Beliefs are “pessimistic” if, when an equilibrium where government bonds trade at a premium exists, investors coordinate on that equilibrium. Finally, in a regime of “extreme” beliefs, investors are willing to finance the government only at risk-free rates, i.e., provided the government is not expected to default in any circumstances. The distinction between “pessimistic” and “extreme” beliefs allows us to study loss of market access at high and low levels of debt, respectively.

Our main results are as follows. First, in addition to slow-moving debt crises, two types of fast ones, in the form of rollover crises, are pervasive in models adopting a dynamic setting after [Calvo \(1988\)](#). When investors coordinate on extreme beliefs, our model comes close to reproduce the dynamics of fast crises in [Cole and Kehoe \(2000\)](#). As a contribution to the debate in the literature, however, we underscore that rollover crises are also possible when the regime of investors’ expectations turns from optimistic to pessimistic, at sufficiently high levels of debt. Conditional on this switch, sudden stops occur when investors realize that there is no positive bond price that simultaneously satisfies the equilibrium pricing conditions and the equilibrium government financing need. When this is the case, the government loses market access.

Rollover crises under pessimistic and extreme beliefs have different quantitative implications. In a numerical example using our baseline model, we show that, under pessimistic beliefs, rollover (fast) crises are possible at high levels of debt—between 122% and 206% of GDP, as opposed to slow-moving crises, which are possible at intermediate levels of initial debt—between 72% and 122% of GDP. Under extreme beliefs, the debt threshold at which rollover crises can occur is instead very low—in our numerical example, well below 40% of GDP. If and when investors coordinate their beliefs on these regimes, the market for government bonds (prior to the crisis traded at the riskless price) may suddenly disappear at either low or high levels of debt, while at intermediate levels, shifts in expectations may raise borrowing costs and ignite high rates of debt accumulation.

Second, distinct from [Lorenzoni and Werning \(2019\)](#), lengthening the maturity of government debt per se does not rule out equilibrium multiplicity leading to slow-moving debt crises—these remain pervasive for *all debt maturities* in both our baseline and in a version of our model close to the debt-limit framework adopted by [Lorenzoni and Werning \(2019\)](#). However, longer maturities may rule out fast rollover crises. In a debt-limit framework, this is the case when, in addition to a long debt maturity, the probability of a recovery is non-negligible. In our baseline model, the parameter restrictions for ruling out fast crises are much more stringent.

Third, in a sunspot equilibrium where switching from the good to a bad equilibrium is

assigned a positive, arbitrarily small probability, the government may have an incentive to deleverage, even during recessions, to bring and keep debt below the crisis threshold. This incentive is strong when the relevant switch is from optimistic to extreme beliefs. As is the case in [Cole and Kehoe \(2000\)](#), the prospect of a rollover crisis leads the government to practice precautionary austerity, and reduce the debt level running pro-cyclical deficit policies. Different from [Cole and Kehoe \(2000\)](#), however, we show that the model predictions are more nuanced when investors and policymakers are concerned with a regime switch from optimistic to pessimistic beliefs. In this case, policymakers find it optimal to deleverage only when the debt level is close enough to the debt threshold at which belief-driven slow-moving debt crises can no longer occur. In a recession, for debt levels sufficiently higher than such thresholds, the consumption smoothing motive dominates fiscal policy, causing deficits and debt accumulation—i.e., gambling on the recovery.

Fourth, we show that short debt maturities induce more deleveraging—a result that resonates with the analysis of debt dilution by [Aguiar and Amador \(2020\)](#). When the maturity of debt is short, the government fully appropriates the benefits from saving to reduce borrowing costs. With long-term debt, instead, the gains from lower borrowing costs are shared between bond holders (as a capital gain) and the government (higher price for new issuance). In line with these considerations, our model suggests that the government finds deleveraging less attractive when debt is long term.

From a policy perspective, our analysis has key implications for debt sustainability analysis and the design of policies to enhance sustainability. First, estimates of debt tolerance thresholds are a crucial input in assessing the extent to which a country can steer away from default. Our results reiterate that these thresholds are not only contingent on the current and future state of the economy and/or preferences of the policymakers. They can also be quite sensitive to a variety of investors’ beliefs. This consideration is a challenge to debt sustainability analysis, motivating an investment in sharpening the analytical toolkit employed in the assessment exercises.

Second, our result that a long debt maturity is not a cure-all solution to the problem of multiple equilibria warns against betting solely on debt management strategies rebalancing the debt maturity structure. By the same token, our result that bond prices may not be falling or move with sunspot probability stresses that market prices may not be reliable signals to detect prospective contingent crises.

Last but not least, our results show that pervasive crisis risk may not provide enough of a welfare incentive for implementing (even optimally smoothed) debt reduction strategies. Indeed, our model provides a key benchmark against which to assess political economy factors, e.g., the role of short-sighted or self-interested policymaking. In our framework, even a forward-looking benevolent government generally finds it optimal to raise debt in a

recession, smoothing consumption at the cost of keeping the country in a state of vulnerability to self-fulfilling crises. Specifically, deleveraging is optimal only at relatively low levels of debt—once the stock of liabilities is large enough, the smoothing motive dominates. Because of cross-border financial contagion, this result strengthens the case for an international compact, where the challenge is how to design official assistance combining liquidity and official loans, to favor economically and politically acceptable policies of deleveraging—essentially reducing the costs of adjustment while creating incentives to implement adjustment policies.

The literature. This paper draws on the seminal contributions by [Calvo \(1988\)](#) and [Conesa and Kehoe \(2017\)](#), in turn related to [Cole and Kehoe \(2000\)](#). [Calvo \(1988\)](#) introduced the feedback loop between self-fulfilling expectations and debt burdens in a two-period model, where the government financing need is taken as given, and the price and quantity of bonds are jointly determined in equilibrium. Self-fulfilling expectations of default generate market “runs” that manifest themselves in a surge in the interest rate charged by investors to the government—but no rollover crisis is modelled in the same context. Conversely, [Conesa and Kehoe \(2017\)](#) focus on liquidity crises whereby the market may suddenly become unwilling to roll over government debt in anticipation of a default. In our paper we aim to reconsider the nature and dynamics of rollover crises—we do so by specifying a model in the style of [Calvo \(1988\)](#) model, but adopting a dynamic setting using the same environment as [Conesa and Kehoe \(2017\)](#) except for the specification of auctions underlying their view of rollover crises.

It is virtually impossible to provide a fair account of the rich literature on debt crises that has contributed to these two paradigms, directly and indirectly.³ [Lorenzoni and Werning \(2019\)](#) reconsider [Calvo \(1988\)](#) in a dynamic setting, stressing that the increase in the sovereign’s borrowing costs driven by self-fulfilling expectations of default leads a country to accumulate debt slowly but relentlessly over time. As the debt stock rises, at some point default occurs unless the conditions of the economy improve sufficiently. [Ayres et al. \(2018\)](#) adopt a framework similar to [Arellano \(2008\)](#) but for the timing assumption, to investigate the likelihood that a country becomes vulnerable to belief-driven crises. Also drawing on [Calvo \(1988\)](#), [Corsetti and Dedola \(2016\)](#) and [Bacchetta et al. \(2018\)](#) write monetary models and discuss how the central bank can backstop government debt, i.e. eliminate self-fulfilling crises by using, respectively, either unconventional (balance sheet) policy, or conventional

³A partial list includes [Alesina et al. \(1989\)](#), [Aguiar and Gopinath \(2006\)](#), [Chamon \(2007\)](#), [Yue \(2010\)](#), [Grauwe \(2011\)](#), [Chatterjee and Eyigungor \(2012\)](#), [Mendoza and Yue \(2012\)](#), [Arellano and Ramanarayanan \(2012\)](#), [Araujo et al. \(2013\)](#), [Cruces and Trebesch \(2013\)](#), [Ghosh et al. \(2013\)](#), [Lizarazo \(2013\)](#), the Handbook chapter by [Aguiar and Amador \(2014\)](#) and [Aguiar et al. \(2016\)](#), [Aguiar et al. \(2015\)](#), [Collard et al. \(2015\)](#), [Tirole \(2015\)](#), [Chatterjee et al. \(2016\)](#), [Bianchi et al. \(2018\)](#), [Ayres et al. \(2019\)](#), [Bocola et al. \(2019\)](#), [Chatterjee and Eyigungor \(2019\)](#), [Galli \(2019\)](#), [Paluszynski \(2019\)](#), and [Stangebye \(2020\)](#).

(inflation) policy.⁴

Several papers have developed the model with rollover crises of [Cole and Kehoe \(2000\)](#), in new directions. By way of example, [Bocola and Dovis \(2019\)](#) characterize how the maturity of sovereign debt can be structured to respond to rollover risk and fundamental risk. Rollover crises are also modelled and discussed by [Giavazzi and Pagano \(1989\)](#), [Alesina et al. \(1992\)](#) and [Cole and Kehoe \(1996\)](#). The importance of liquidity lending for a currency union is emphasized by [Aguiar et al. \(2015\)](#), suggesting that the co-existence of high debt and low debt countries in a currency area may create incentive for liquidity provisions that benefit also relatively virtuous countries—and the sustainability of the area overall.

The goal of our paper is close to [Aguiar et al. \(2020\)](#), who also address the need to develop a unified framework to account for the variety of crises that we observe in the data. To pursue this goal, [Aguiar et al. \(2020\)](#) enrich [Cole and Kehoe \(2000\)](#) allowing for uncertainty about the default decision by the government once the debt auction is closed. The new model specification creates the possibility of belief-driven hikes in borrowing rates conceptually similar to the one stressed by [Calvo \(1988\)](#), however occurring in what these authors dub a “static” dimension (with inter-temporal implications). Our work is clearly complementary to theirs, as we essentially pursue a very similar goal building on [Calvo \(1988\)](#).

This paper is organized as follows. Section 2 lays out the model similar to [Conesa and Kehoe \(2017\)](#) but with a different timing assumption. For our baseline, Section 3 discusses “static” equilibrium multiplicity with different types of beliefs, while Section 4 presents a calibrated numerical example of Section 3 with long-term debt. Section 5 analyzes whether the perceived threat of a belief-driven crisis would prevent a government from running deficits during recessions. Section 6 carries out sensitivity analysis of our baseline and a debt-limit model focusing on debt maturities and probabilities of recovery. Section 7 concludes.

2 Model

In this section we specify our dynamic model of debt sustainability and default.⁵ The environment draws on [Conesa and Kehoe \(2017\)](#), except that we set the timing of investors’ and the government decisions in the style of models after [Calvo \(1988\)](#), such that a government would not be able to set the amount of bonds to be issued before investors set the bond price.

⁴See also [Aguiar et al. \(2013\)](#).

⁵In writing this paper we draw extensively on previous work on debt bailout, especially on [Corsetti et al. \(2017\)](#), which introduces official lending in a [Conesa and Kehoe \(2017\)](#) framework, but also on [Corsetti et al. \(2020\)](#), [Conesa and Kehoe \(2014\)](#), [Roch and Uhlig \(2018\)](#) and [Marin \(2017\)](#).

2.1 Environment

We consider an environment populated by a large number of investors, a government, and the representative household. Time is discrete and indexed by $t = 0, 1, 2, \dots$. To restrict our attention to the sovereign’s behaviour, we assume that every period the representative household consumes all its income after paying tax.

The country’s endowment follows the function $y(a, z) = A^{1-a}Z^{1-z}\bar{y}$, with $A < 1$ and $Z < 1$. The parameter a indicates whether the economy is in a recession $a = 0$ or not $a = 1$. z denotes the government decision to default $z = 0$ or repay $z = 1$. If the government defaults, it stays at the state of default $z = 0$ forever and the productivity permanently drops by the factor Z . The economy starts out with $a_0 = 0$ and $z_0 = 1$. From period 1, the economy recovers with probability $p < 1$ and once recovered, it never falls back to a recession again.⁶

The government issues non-contingent bonds to risk neutral investors. As is customary after [Hatchondo and Martinez \(2009\)](#), we model the maturity of government bonds as follows. Bonds have geometrically decreasing coupons: a bond issued at t pays the sequence of coupons

$$\kappa, (1 - \delta)\kappa, (1 - \delta)^2\kappa, \dots$$

where $\delta \in [0, 1]$. Hence, assuming risk neutral investors whose discount factor is β , the price of a default-risk-free bond is

$$q = \frac{\beta\kappa}{1 - \beta(1 - \delta)}$$

To normalize bond prices, it is convenient to set $\kappa = 1 - \beta + \beta\delta$ so that the price of a default-free bond is β . The parameter δ indexes the maturity of debt, where $\delta = 0$ corresponds to the case of “consols” (or perpetuities) and $\delta = 1$ corresponds to the case of short-term bonds. Note that a bond issued at $t - m$ is equivalent to $(1 - \delta)^m$ bonds issued at t . Hence, the stock of outstanding bonds can be summarized by a single state variable B .

Throughout our analysis, we will assume that investors can coordinate on different regimes of beliefs. Following the literature, coordination is driven by an exogenous state ρ —which, when the equilibrium is not unique, determines which one is selected. The aggregate state variable of the economy is then summarized by $s = \{(B, z_{-1}, a, \rho)\}$.

⁶The model can be extended to adopt the bimodal income process used by [Ayres et al. \(2018\)](#), [Chatterjee and Eyigungor \(2019\)](#), [Ayres et al. \(2019\)](#) and [Paluszynski \(2019\)](#). In such a setting, there are equilibria located at the “wrong” side of the debt Laffer curve, in which a small increase in the bond price creates extra demand of bonds. For the sake of clarity and simplicity, we adopt the discrete distribution such that the debt Laffer curve is locally increasing at all points to filter unstable equilibria without the need to impose an equilibrium selection criterion. Equilibria presented in our model are in line with stable equilibria from a bimodal income process.

2.2 Timing

The timing of debt issuance is the same as [Calvo \(1988\)](#).⁷ The timeline below is written under the maintained assumption that, if the government has defaulted in the past ($z_{-1} = 0$), it is punished by markets and denied market access in the current and future periods—it can only operate under financial autarky.

1. The shock a is realized. An exogenous beliefs state ρ is also realized—we elaborate on beliefs in [Section 3](#). The aggregate state $s = \{(B, z_{-1}, a, \rho)\}$ is known.
2. Risk neutral investors set the bond price $q(b', s)$, which may be zero, consistent with their planned individual purchase of debt b' .
3. Fiscal/default decision takes place:
 - Provided no default has occurred in the previous period ($z_{-1} = 1$), taking the bond price as given, the government decides whether to default or repay, adjusting its deficit optimally to the current bond prices.
4. Equilibrium allocations are realized:
 - If the government decides to default (or has already defaulted in the past), its output drops by the factor Z and it loses market access, both on a permanent basis.
 - If the government repays, it chooses its primary deficit and thus its bond issuance B' , and the bond market clears $B' = b'$ at the current positive equilibrium bond price.

A comment is in order concerning the difference between our timing assumption and the assumption by the literature after [Eaton and Gersovitz \(1981\)](#) and [Cole and Kehoe \(2000\)](#). In this literature, the government sets the total issuance of bonds at face value first; investors set bond prices afterwards. In [Cole and Kehoe \(2000\)](#), this timing assumption acts as a selection criterion that rules out crises of the type analyzed by [Calvo \(1988\)](#).⁸ Implicit in this assumption is that, once the government commits to issue a predetermined amount of bonds at some anticipated equilibrium price, it must be able to adjust its primary surplus to make up for any shortfall in fiscal revenue if the equilibrium bond price turns out to be lower than anticipated. The point of departure of our analysis is to relax this assumption, and allow

⁷[Corsetti and Dedola \(2016\)](#) and [Lorenzoni and Werning \(2019\)](#) adopt the same timing assumption.

⁸In this model, provided investors are willing to finance the government, they offer the best equilibrium price for the bonds. [Auclert and Rognlie \(2016\)](#) expand on [Eaton and Gersovitz \(1981\)](#) and discuss conditions such that a unique equilibrium exists.

the government to adjust both its fiscal surplus and its bond issuance to market conditions—that is, vis-à-vis the equilibrium bond prices offered by investors $q(b', s)$ at stage 2. In line with the arguments by [Lorenzoni and Werning \(2019\)](#), we find this timing assumption a plausible working hypothesis.

2.3 The Investors' Problem

For tractability, we follow the literature and assume that investors are perfectly competitive and risk neutral with discount factor β . They have “deep pockets” such that the corner solutions in each investor’s problem are ruled out in equilibrium. The equilibrium condition from the investors must satisfy the break-even condition that equates the expected return on sovereign debt to the risk-free rate:

$$q(b', s) = \begin{cases} \beta \mathbb{E} \left[z(s') \left(\kappa + (1 - \delta) q(b'(s'), s') \right) \right] & \text{if } z_{-1} = 1 \\ 0 & \text{if } z_{-1} = 0 \end{cases} \quad (1)$$

If the government has no history of default in the past⁹, the equilibrium price depends on the probability of defaulting in the future.

The probability of defaulting may vary with the equilibrium on which investors coordinate their expectations/beliefs—coordination being driven by an exogenous beliefs state ρ . Given our timing assumption, these investors’ beliefs affect the equilibrium debt issuance and the debt tolerance thresholds that reflect optimal contingent default decisions by the government.

2.4 The Government's Problem

In our economy, in each period the government can access the bond market only conditional on a good credit history, and takes its fiscal decision conditional the equilibrium bond price schedule $q(B', s)$ offered by atomistic and deep-pocketed investors. Given this bond price schedule, and assuming for simplicity that the tax rate τ is constant (so that the sovereign tax revenue is exogenous and equal to $\tau y(a, z)$), the government problem can be reduced to choose B', z to solve

$$\begin{aligned} V(s) = \max_{B', z} \quad & \mathcal{U}(c, g) + \beta \mathbb{E}[V(s')] \\ \text{s.t.} \quad & g + z\kappa = \tau y(a, z) + zq(B', s)(B' - (1 - \delta)B), \\ & c = (1 - \tau)y(a, z), \\ & z = 0 \text{ if } z_{-1} = 0 \end{aligned} \quad (2)$$

⁹Recall that, once the government defaults ($z = 0$), z stays at 0 forever. This assumption implies that the equilibrium bond price is zero in any history with past default.

where c represents the consumption of the representative household who spends all its income after paying tax every period. We denote the endogenous government spending as g , and we stipulate that there is some critical expenditure level \bar{g} , below which the normal functioning of the state becomes problematic.

In the framework presented above, the government defaults if and only if the utility of repaying debt $V^R(s)$ is smaller than the utility of defaulting $V^D(s)$:

$$V^R(s) < V^D(s)$$

The value of defaulting is determined by assuming that, in case of debt repudiation, the country loses market access *and* experiences a discrete contraction in output by Z —output stays at $AZ\bar{y}$ in a recession and $Z\bar{y}$ in normal times. Both costs are assumed to be permanent. This defines our baseline default model.

As in [Conesa and Kehoe \(2017\)](#), we posit that, for any feasible B such that $\tau A\bar{y} - B$ is an element of the feasible set of government spending g , the following condition holds

$$\mathcal{U}_g((1 - \tau)A\bar{y}, \tau A\bar{y} - B) > \mathcal{U}_g((1 - \tau)\bar{y}, \tau\bar{y} - B)$$

This ensures that, in a recession, the government always has an incentive to raise debt due to higher marginal benefit of government spending in a recession than in normal times.

2.5 Equilibrium

An equilibrium is a value function for the government $V(s)$ and policy functions $B'(s)$, $z(s)$ and $g(s)$, and an equilibrium bond price schedule $q(B', s)$ such that

1. Given policy function $z(s)$, $g(s)$, $V(s)$ and $B'(s)$, $q(b', s)$ is such that investors make zero expected profit (the break-even condition [\(1\)](#)), and $q(B', s)$ is consistent with market clearing and rational expectations.
2. $V(s)$, $B'(s)$, $z(s)$ and $g(s)$ solve government optimization problem in [\(2\)](#) given the bond price function $q(B', s)$.

For tractability, the notion of equilibrium we consider follows a simple Markov structure.

3 Equilibrium Multiplicity with Short-Term Debt

To illustrate the logic and consequences of equilibrium multiplicity, we specialize the baseline model laid out above assuming that debt is short-term only, and all agents in the economy consider investors' beliefs as constant over time. The equilibrium corresponds to what [Aguiar](#)

et al. (2020) dubbed “static”, in the sense that, once an exogenous beliefs state ρ determines which regime of expectations prevails in the market, agents do not expect to switch across regimes. In other words, the model is solved condition on one regime of beliefs only. A switch, if it occurs, is completely unanticipated. We relax this assumption later on in the text.

In what follows, we begin our analysis defining and discuss the set of beliefs that are relevant in our model. Consistent with the solution of the model, we focus on a three-element set. Namely we let $\rho \in \{O, P, E\}$, distinguishing between O for “Optimistic”, P for “Pessimistic”, and E for “Extreme” beliefs.

3.1 Beliefs: Optimistic, Pessimistic and Extreme

We find it useful to start our analysis with a heuristic description of how beliefs come into play in affecting pricing decision and the equilibrium allocation—taking the set of possible equilibria as given. In each of these equilibria, one-period bonds are traded at either the riskless or the risky price, which under risk neutrality are, respectively, β and βp , or are not traded at all.

Optimistic Beliefs

In a regime of optimistic beliefs, when more than one equilibrium is possible, investors always coordinate their expectations on the one with the best price for government bonds. Intuitively, one can envision investors approaching the market assuming that government bonds are safe. Given the state of the economy and the size of outstanding debt, the riskless price may/may not be self-validating in equilibrium. If it is, optimistic investors go ahead and offer the riskless price. Otherwise, they revise their assessment and focus on an equilibrium with bonds traded at the risky price. Again, this may/may not be possible. If not, they simply refuse to buy the government debt (the equilibrium price is zero) and the government loses market access.

Pessimistic Beliefs

In a regime of pessimistic beliefs, investors are generically willing to finance the government, but coordinate their expectations on equilibria where the government bonds trade at the default-risky price βp —they assume that the government will repay existing obligations if and only if the economy recovers in the following period. This equilibrium may/may not exist—i.e., the risky price may/may not be self-validating. If it is not, they reconsider the economic conditions. These may be so favorable that they purchase bonds at the riskless

price—or so bad that they refuse to finance the government altogether.¹⁰

Extreme Beliefs

As a reference benchmark, we also consider the possibility that investors hold what we dub “extreme beliefs”. Intuitively, we can think that, holding extreme beliefs, investors are only willing to finance the government in circumstances under which there is no default in equilibrium. For instance, in our economy with short-term debt, beliefs are extreme when investors anticipate the government to systematically default *next period* (hence are not willing to pay any positive price for its short-term bonds), unless debt is so low that the government can be expected to honour its liabilities *today* and *tomorrow*. We will see that, while the precise mechanism by which our extreme beliefs may lead to rollover crises is different from [Cole and Kehoe \(2000\)](#), this case will come into handy in comparing our model with their seminal work.

Under extreme beliefs, the utility of repaying debt V_E^R solves

$$V_E^R(s) = \max_{0 \leq B'} \mathcal{U}((1 - \tau)y(a, z), \tau y(a, z) - B + 0 \times B') + \beta \mathbb{E}[V(s')|s] \quad (3)$$

where, given Calvo timing, the loss of market access for the government coincides with markets coordinating on a 0 bond market price. When $V_E^R(s) \geq V^D(s)$, the government does not default *today* and it will not default *next period*. As a result, $q = 0$ is not an equilibrium price and “extreme beliefs” are not validated. In contrast, when $V_E^R(s) < V^D(s)$, the government defaults immediately and, by the assumption that defaulting governments are punished by investors via market exclusion, the loss of market access implied by the investors’ extreme beliefs is validated in equilibrium. In other words, in the model, these beliefs are subject to the primitive assumption that the government *permanently* stays at the state of default and market exclusion in any history of default. As mentioned above, however, these beliefs may be rationalized as a strict solvency requirement.¹¹

3.2 The Debt Tolerance Thresholds

Debt tolerance thresholds are contingent on the state of the economy—in our notation, we write $\bar{B}(a)$ $a = 0, 1$: $\bar{B}(0)$ is the maximum sustainable debt level in a recession and $\bar{B}(1)$

¹⁰We should note that, under our simplifying assumption, once the economy recovers, the default-risky price is no longer an equilibrium price, in that the output uncertainty is resolved. Without loss of generality, we posit that investors offer the riskless bond price after the recovery.

¹¹In [Cole and Kehoe \(2000\)](#) and [Conesa and Kehoe \(2017\)](#), the utility off the equilibrium path in rollover crises is exactly the same as (3)—the auction failure mechanism is the key to induce rollover crisis. By contrast, we show that Calvo timing can reproduce rollover crisis similar to [Cole and Kehoe \(2000\)](#) when adopting the assumption that the economy stays at the state of default forever in any history of default. We elaborate the differences of various rollover crises in detail in Section 3.3

is the maximum sustainable debt level in normal times. Without loss of generality, in what follows we posit that the government has not defaulted in the past $z_{-1} = 1$.

3.2.1 The debt tolerance threshold in normal times, $\bar{B}(1)$.

Since we assume that, after the economy recovers, a stays at 1 forever, the government optimization problem is deterministic for beliefs that are either optimistic or pessimistic ($\rho = O$ or P). Conditional on deciding to repay its debt, the government pays $(1 - \beta)B$ in each period to satisfy the no-Ponzi condition. Let $V^R(B, a, \rho)$ and $V^D(a)$ denote the government utility of, respectively, repaying debt, and defaulting:

$$V^R(B, 1, \rho = O \text{ or } P) = \frac{\mathcal{U}((1 - \tau)\bar{y}, \tau\bar{y} - (1 - \beta)B)}{1 - \beta}$$

The utility of defaulting when the economy is not in a recession is

$$V^D(1) = \frac{\mathcal{U}((1 - \tau)Z\bar{y}, \tau Z\bar{y})}{1 - \beta}$$

It follows that $\bar{B}(1)$ can be characterized by solving

$$V^R(\bar{B}(1), 1, \rho = O \text{ or } P) = V^D(1)$$

and that the utility of the government $V(B, a, \rho)$ in normal times is summarized by

$$V(B, 1, \rho = O \text{ or } P) = \max \left\{ V^R(B, 1, \rho = O \text{ or } P), V^D(1) \right\}$$

A different threshold follows from extreme beliefs, on which we will expand in our analysis of sunspot equilibria in Section 5.2.¹²

The debt tolerance threshold $\bar{B}(1)$ coincides under the optimistic/pessimistic beliefs regime, since investors hold a unique consistent view that the economy will never fall back into a recession—output uncertainty is resolved. By contrast, when the economy is in a recession, fundamental risk drives debt tolerance thresholds apart under different regimes of beliefs, as we show next.

3.2.2 The debt tolerance threshold(s) in a recession $\bar{B}(0)$.

We now discuss the derivation of $\bar{B}(0)$ for each beliefs regime ρ —denoting with $\bar{B}(0)_{opt}$, $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{EX}$ the thresholds for optimistic, pessimistic and extreme beliefs, respectively.

¹²In that section, we will use the debt threshold conditional on extreme beliefs ($\rho = E$), denoted by $\bar{B}(1)_{EX}$. In accordance with (3), $\bar{B}(1)_{EX}$ is pinned down by solving the following equation: $\mathcal{U}((1 - \tau)\bar{y}, \tau\bar{y} - \bar{B}(1)_{EX}) + \beta \frac{\mathcal{U}((1 - \tau)\bar{y}, \tau\bar{y})}{1 - \beta} = V^D(1)$, unambiguously smaller than $\bar{B}(1)$ ($\rho = O$ or P). Conditional on extreme beliefs, a collapse of bond market may lead to an immediate default regardless of the output states, as long as it validates the view that the government will be at the state of default next period.

Optimistic Beliefs

When investors hold optimistic beliefs, they presume that the government may remain willing and able to service its debt in full even if the economy remains in a recession in the next period. The question is whether this presumption is self-validating in equilibrium—if not, they could still lend to the government at the default-risky price βp . To verify this, it is important to allow for the possibility that the debt issuance capacity of the government is constrained by a debt threshold below the one conditional on the economic recovery $\bar{B}(1)$. While the utility of defaulting in a recession is independent of beliefs:

$$V^D(0) = \frac{\mathcal{U}((1-\tau)AZ\bar{y}, \tau AZ\bar{y})}{1 - \beta(1-p)} + \beta \frac{p\mathcal{U}((1-\tau)Z\bar{y}, \tau Z\bar{y})}{(1-\beta)(1-\beta+\beta p)}$$

the utility of repaying debt is belief-dependent. Writing the utility of repaying debt contingent on optimistic beliefs $V_O^R(B, 0) = \max\{V_{O,1}^R(B, 0), V_{O,2}^R(B, 0)\}$, either of the following may be relevant:

$$\begin{aligned} V_{O,1}^R(B, 0) = \max_{0 \leq B' \leq \bar{B}(0)_{opt}} & \quad \mathcal{U}(c, g) + \beta \left(pV(B', 1, \rho = O) + (1-p)V(B', 0, \rho = O) \right) \\ \text{s.t.} & \quad g + B = \tau A\bar{y} + \beta B', \\ & \quad c = (1-\tau)A\bar{y} \end{aligned} \tag{4}$$

$$\begin{aligned} V_{O,2}^R(B, 0) = \max_{\bar{B}(0)_{opt} < B' \leq \bar{B}(1)} & \quad \mathcal{U}(c, g) + \beta \left(pV(B', 1, \rho = O) + (1-p)V^D(0) \right) \\ \text{s.t.} & \quad g + B = \tau A\bar{y} + \beta pB', \\ & \quad c = (1-\tau)A\bar{y} \end{aligned} \tag{5}$$

The debt threshold $\bar{B}(0)_{opt}$ is the solution of the equation below.

$$V_O^R(\bar{B}(0)_{opt}, 0) = V^D(0)$$

Define a set that validates “optimistic belief”:

$$\mathbb{B}_{opt} \equiv \{B \mid V_O^R(B, 0) \geq V^D(0)\}$$

On this domain, $V(B, 0, \rho = O) = V_O^R(B, 0)$. For $B > \sup \mathbb{B}_{opt} = \bar{B}(0)_{opt}$, the government defaults.

Pessimistic Beliefs

When investors hold pessimistic beliefs, given the equilibrium financing need of the government, they are mainly concerned with the default in the following period if a recession persists—a scenario that they assess conditional on setting the bond price at βp . At this

price, the utility of repaying debt $V_P^R(B, 0)$ can be characterized by (4) and (5), provided the default-free bond price β in (4) is replaced with βp . To save space, we write up a full description of $V_P^R(B, 0)$ in Appendix A.

The debt threshold $\bar{B}(0)_{pes}$ at which the risky bond price is an equilibrium solves the equation below:

$$V_P^R(\bar{B}(0)_{pes}, 0) = V^D(0)$$

A necessary condition of self-validated pessimistic beliefs is $B' > \bar{B}(0)_{pes}$ in $V_P^R(B, 0)$, so to ensure that the government defaults when a recession persists. Define \mathbb{B}_{pes} as the set of initial debt levels that validate “pessimistic belief”:

$$\mathbb{B}_{pes} \equiv \{B \mid B' \text{ that solves } V_P^R(B, 0) \text{ satisfies } B' > \bar{B}(0)_{pes} \text{ and } V_P^R(B, 0) \geq V^D(0)\}$$

The following proposition establishes that \mathbb{B}_{pes} is always non-empty.

Proposition 1. *For a strictly concave utility function \mathcal{U} , a sufficient condition for $\mathbb{B}_{pes} \neq \emptyset$ is a high enough critical expenditure \bar{g} , such that under the pessimistic beliefs regime, there exists some debt level at which the government is unable to sustain \bar{g} unless it issues debt above $\bar{B}(0)_{pes}$.*

Proof. See Appendix C.1. □

Proposition 1 proves the existence of equilibria given pessimistic beliefs. Different from the original analysis in Calvo (1988), where the low bond price solution lies at “wrong” side of the debt Laffer curve $q(B', s)B'$, the equilibrium when $\rho = P$ in our model is “stable” in the sense discussed by Lorenzoni and Werning (2019). This is apparent in the figures below, see Figure 1-3, where the equilibrium points on the Laffer curve are all locally increasing and a small deviation from the equilibrium does not create any instability.¹³

For $B \in \mathbb{B}_{pes}$, the utility of the government in a recession is $V(B, 0, \rho = P) = V_P^R(B, 0)$. If initial outstanding debt level $B < \inf \mathbb{B}_{pes}$, even if investors hold the pessimistic belief, the only equilibrium that can justify investors’ break-even condition (1) is the risk-free price β . When $B > \sup \mathbb{B}_{pes} = \bar{B}(0)_{pes}$, the government defaults immediately. We elaborate these two cases in detail in Section 3.3.

Extreme Beliefs

When investors hold extreme beliefs, they are not willing to finance the government unless default can be excluded in all circumstances—which include the possibility of a loss of market access one period ahead. Following the formula (3), the utility of repaying debt is

$$V_E^R(B, 0) = \mathcal{U}((1 - \tau)A\bar{y}, \tau A\bar{y} - B) + \beta \left(pV(0, 1, \rho = E) + (1 - p)V(0, 0, \rho = E) \right) \quad (6)$$

¹³The endowment process in our model can be viewed as drawn from a bimodal distribution as in Ayres et al. (2018), Ayres et al. (2019), Chatterjee and Eyigungor (2019) and Paluszynski (2019).

The debt threshold $\bar{B}(0)_{EX}$ can be characterized by solving

$$V_E^R(\bar{B}(0)_{EX}, 0) = V^D(0)$$

We define the set of initial debt level \mathbb{B}_{EX} that validates “extreme belief”:

$$\mathbb{B}_{EX} \equiv \{B \mid V_E^R(B, 0) < V^D(0)\}$$

The following proposition shows that $\bar{B}(0)_{EX}$ is always positive:

Proposition 2. *A strictly concave utility function with the property $\lim_{g \rightarrow \bar{g}} \mathcal{U}(c, g) = -\infty$ ensures $\bar{B}(0)_{EX}$ is positive.*

Proof. See Appendix C.2. □

Proposition 2 proves the existence of equilibrium conditional on extreme beliefs. When $B > \bar{B}(0)_{EX}$ and $\rho = E$, the bond price is 0 and the government, facing a loss of market access, defaults immediately. In what follows, we will show that a rollover crisis may also occur above the debt threshold under “pessimistic” beliefs. Specifically, when $B > \bar{B}(0)_{pes}$ and $\rho = P$, the government defaults immediately and the bond price is zero. We will compare and contrast the ‘fast’ crises corresponding to different regimes of beliefs, as they feature a high degree of similarity, but distinct equilibrium mechanisms and the economic interpretation. These differences will be apparent from our graphical analysis in the next subsection.

The algorithm for computing an equilibrium in a recession is shown in Appendix B. To gain insight on the thresholds characterized above, in the next subsection we rely on a simple graphical apparatus.

3.3 An Intuitive Graphical Analysis

Multiple equilibria may exist depending on outstanding debt level. In Figure 1-3, we study graphically equilibria for three different levels of initial debt—low, intermediate and high. Each figure includes two panels—the left panel depicting the equilibrium under optimistic beliefs ($\rho = O$), the right hand panel depicting the equilibrium under pessimistic beliefs ($\rho = P$). To avoid complicating the graphs, we omit the equilibrium under extreme beliefs from the figures. Instead, we discuss the differences of rollover crises driven by pessimistic and extreme beliefs at the end of this subsection.

In each figure, the x axis measures the amount of bonds the government issues during the period, the y axis measures the resources that the government can obtain by issuing debt at the equilibrium price, qB' . Vertical dashed lines denote the debt tolerance thresholds derived above. The solid (blue, discontinuous) line in the left panel depicts the Laffer curve

conditional on optimistic beliefs; the solid (red discontinuous) line depicts the Laffer curve in the right panel conditional on pessimistic beliefs. From the origin to the threshold $\bar{B}(0)$, the Laffer curve has slope β , the risk-free bond price. Beyond this threshold, the price of debt falls as investors anticipate contingent default one period ahead. Due to default risk, any issuance bringing the debt stock in the range between $\bar{B}(0)$ and $\bar{B}(1)$ is priced βp : the Laffer curve has a flatter slope (still upward sloping).

The dynamics of issuance are driven by the Gross Financing Need of the government (GFN):

$$qB' = g + \underbrace{B - \tau A\bar{y}}_{GFN(q;B)}$$

Since the government chooses spending, the GFN is endogenous, as a function of the price of bonds and the initial debt. Because of this endogeneity, the GFN draws a downward sloping line in our graph: over the relevant range of debt levels, at lower bond prices (higher interest rates) the government optimally reduces its primary deficit. Intuitively, moving down the GFN line, think of each point as crossing a ray from the origin (not shown), corresponding to a lower bond price q . At lower q 's, the government faces higher borrowing costs. The government has thus an incentive to adjust its spending optimally, reducing its financing need. However, this optimal adjustment falls short of reducing the new issuance of debt, hence the GFN line is decreasing monotonically. The initial level of debt instead determines the position of the GFN in the graph. A higher stock of liabilities inherited by the government moves the GFN out to the right.

In the figures, the debt tolerance thresholds are depicted in line of the following result:

Proposition 3. $\bar{B}(0)_{opt} \geq \bar{B}(0)_{pes}$. *Equality holds if and only if, under optimistic beliefs regime, the government chooses to borrow into a default-risky level when B is equal to $\bar{B}(0)_{opt}$, i.e., B' is above $\bar{B}(0)_{opt}$. Otherwise inequality is strict.*

Proof. See Appendix C.3. □

Without loss of generality, we posit that the government never chooses to issue debt above $\bar{B}(0)_{opt}$ when $\rho = 0$ so to ensure $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$.¹⁴ It is worth stressing that, in the model (and in the graph), the debt tolerance thresholds, hence the Laffer curves, differ across the optimistic and the pessimistic scenarios, but are independent of the initial debt level. Hence the debt thresholds and the Laffer curves (one for the optimistic, the other for the pessimistic beliefs) are exactly the same across Figure 1-3 below.

Figure 1 illustrates a case in which the initial debt stock B_{low} is so low that the equilibrium is unique and bonds are traded at default-free prices. In the left-hand panel, the GFN

¹⁴The parameter restrictions to achieve $\bar{B}(0)_{opt} = \bar{B}(0)_{pes}$ are stringent— $\bar{B}(0)_{opt}$ is generally larger than $\bar{B}(0)_{pes}$ in our baseline. A more general case is discussed in detail in Section 6.

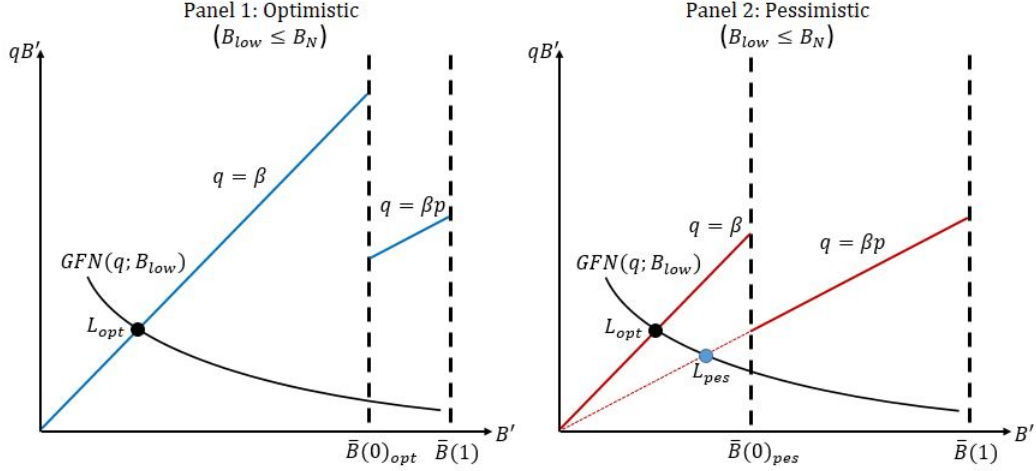


Figure 1: Unique equilibrium when $B_{low} \in [0, B_N]$

intersects the debt Laffer curve at L_{opt} , and debt is issued at risk-free rate. In the right-hand panel, at risky prices, the GFN would imply issuance at the point L_{pes} . It is easy to verify that the latter cannot be an equilibrium. Even if investors confirmed their pessimistic beliefs by offering a low bond price, the government would still issue debt below the tolerance threshold $\bar{B}(0)_{pes}$ since its GFN is moderate. Investors' pessimistic view would not be validated ex-post. The only self-fulfilling equilibrium is at the point L_{opt} , with riskless pricing in equilibrium. In other words, in panel 2, $q = \beta p$ is not the solution to investors' first order condition (1), shown below:

$$\underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E} \left[z \left(\underbrace{B'}_{< \bar{B}(0)}, a', z'_{-1}, \overbrace{\rho = P}^{=1} \right) \right] \quad \bar{B}(0) = \bar{B}(0)_{pes} \quad (7)$$

Note that, using this equation, we can also determine at which initial levels of debt the equilibrium is no longer unique. We find it convenient to define this as a new debt threshold, labelled $B_N \equiv \inf \mathbb{B}_{pes}$, which denotes the maximum amount of the *initial* debt level in a recession below which the country is immune to pessimism. B_N can be found by solving the equation (8) below:

$$B_N = \sup_B \left\{ \underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E} \left[z \left(\underbrace{B'(s)}_{< \bar{B}(0)}, a', z'_{-1}, \overbrace{\rho = P}^{=1} \right) \right] \right\} \quad \bar{B}(0) = \bar{B}(0)_{pes} \quad (8)$$

For initial debt levels larger than B_N , if investors develop a pessimistic view on government solvency, an equilibrium with default can be self-validating: the government either borrows more than $\bar{B}(0)_{pes}$, or defaults immediately. We now turn to these cases, looking at Figure 2 and 3.

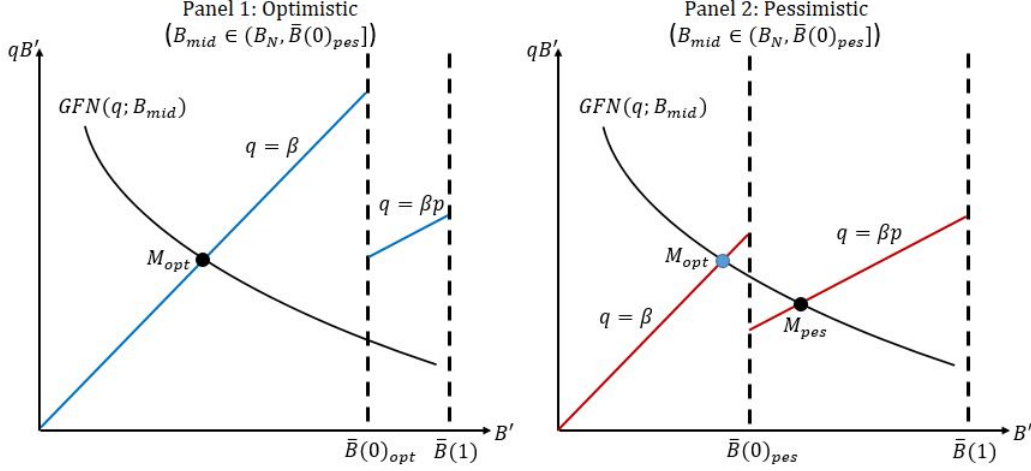


Figure 2: Multiple equilibria when $B_{mid} \in (B_N, \bar{B}(0)_{pes}]$

In Figure 2, the government initial debt B_{mid} is at intermediate level. Precisely, B_{mid} is larger than B_N but not too high—smaller than $\bar{B}(0)_{pes}$. The relevant GFN line intersects the Laffer curves in both panels, at the points M_{opt} and M_{pes} . The former is an equilibrium outcome in an optimistic world, the latter in a pessimistic world. In the panel to the left, when investors buy newly issued sovereign debt at the riskless price β , overall borrowing remains below the relevant debt tolerance threshold $\bar{B}(0)_{opt}$, validating ex-post the investors' optimistic beliefs. By contrast, in the panel to the right, new issuance goes beyond (lower) tolerance threshold $\bar{B}(0)_{pes}$ when investors buy at the lower risky price. One period ahead, unless the economy recovers, the government defaults, validating investors' pessimistic beliefs. Note that, even though the GFN line intersects at M_{opt} in the right panel, M_{opt} is not an equilibrium as investors coordinate their expectations on equilibria where sovereign bonds trade at the default-risky price βp . For both $q = \beta$ and $q = \beta p$ to be equilibrium prices, the initial stock of debt must be such that the equations (9) and (10) from the investors' first order condition (1) are satisfied at once:

$$\underbrace{q(b', s)}_{=\beta} = \beta \mathbb{E} \left[z \left(\underbrace{B'}_{< \bar{B}(0)}, a', z'_{-1}, \underbrace{\rho = O}_{\bar{B}(0) = \bar{B}(0)_{opt}} \right) \right] \quad (9)$$

$$\underbrace{q(b', s)}_{=\beta p} = \beta \mathbb{E} \left[z \left(\underbrace{B'}_{> \bar{B}(0)}, a', z'_{-1}, \underbrace{\rho = P}_{\bar{B}(0) = \bar{B}(0)_{pes}} \right) \right] \quad (10)$$

The type of equilibrium with belief-driven default shown in Figure 2 corresponds to a scenario in which, as stressed by [Lorenzoni and Werning \(2019\)](#), the debt crisis is 'slow-moving'. Interest rates are high because investors expect the government to default if a recession persists. Because of high borrowing costs, the stock of government debt rises prior

to default. But default only occurs if and only if the country remains in a recession in the future.

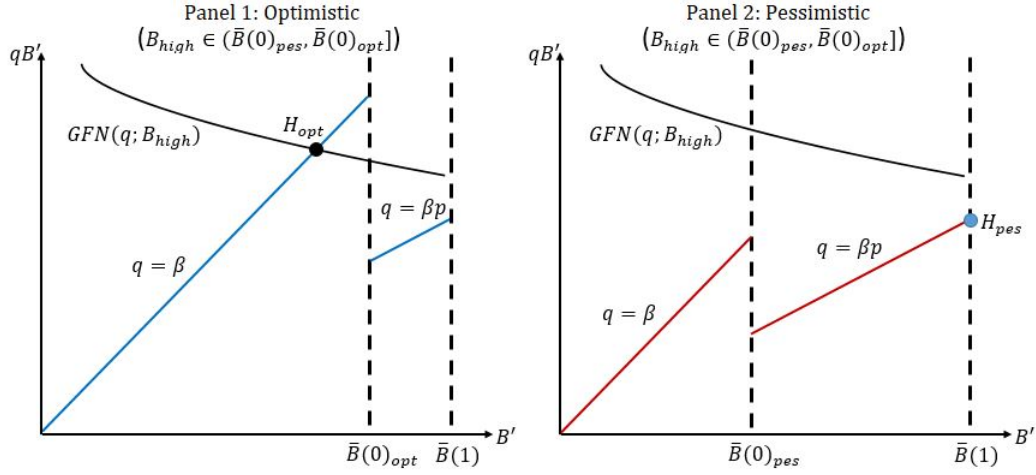


Figure 3: Fast crisis when $B \in (\bar{B}(0)_{pes}, \bar{B}(0)_{opt}]$

Figure 3 illustrates another type of belief-driven default in equilibrium, possible for a relatively high initial debt level, higher than $\bar{B}(0)_{pes}$. In the left-hand side panel, the GFN line now intersects the Laffer curves at the point H_{opt} . If investors buy government bonds at the riskless price β despite the high stock of initial liabilities, new debt issuance remains below the relevant threshold $\bar{B}(0)_{opt}$. However, if investors turn pessimistic—right-hand side panel—the hike in borrowing rates causes the government to become less ‘tolerant’ of the adjustment required to service the debt. Investors anticipate that the government will not be able and/or willing to adjust its primary needs enough to keep new issuance of debt below $\bar{B}(1)$ at the default-risky bond price (the GFN line does not cross the debt Laffer curve). Thus a “fast” debt crisis occurs.

We can look at this “fast” debt crisis from two angles. From the vantage point of the government, when the market expects a default in a recession, new bonds can only be issued at risky rates. But at these rates, even after adjusting its primary surplus, the government is unable to satisfy its financing need keeping debt issuance below its maximum debt capacity in normal times—beyond which a default occurs for sure. Even if investors were willing to charge finite interest rates (presumably conditional on the government cutting its deficit further), immediate default would be the preferred option. Anticipating all these, from the investors’ vantage point, it is rational not to finance the government at all: the country instantly loses market access. In a rollover crisis, the government has no alternative but to

default. The condition is given by equation (11) below.

$$\underbrace{q(b', s)}_{=0} = \beta \mathbb{E} \left[\overbrace{z(B', a', z'_{-1}, \underbrace{\rho = P}_{\bar{B}(0) = \bar{B}(0)_{pes}})}^{=0} \right] \quad (11)$$

It is important to clarify why the point H_{pes} in the figure is not an equilibrium. This is shown in equation (12). Once investors turn pessimistic, the government optimally cuts its deficit and reduces its current financing need, moving down along the GFN line. In principle, the government could implement further cuts in its deficit, but this would never be optimal given that spendings and the utility of the government remain relatively high after default, i.e., given that post default output remains sufficiently high relative to the critical expenditure level \bar{g} .

$$\underbrace{q(b', s)}_{=\beta p} \neq \beta \mathbb{E} \left[\overbrace{z(B', a', z'_{-1}, \underbrace{\rho = P}_{\bar{B}(0) = \bar{B}(0)_{pes}})}^{=0} \right] \quad (12)$$

There is a subtle but important difference between the “fast” debt crisis in Figure 3 and the rollover crisis in Cole and Kehoe (2000). In Cole and Kehoe (2000), if (enough) investors participated in the bond auction, there would be no default. This is because it is precisely the loss of market access that makes the surplus adjustment (required to repay existing obligations) so large and harsh, to be welfare-dominated by default. In Figure 3, instead, the government would default even if investors were willing to finance the deficit at the (off equilibrium) risky price—market access is lost because debt is too high to be sustainable when investors charge a premium.

This difference is best appreciated by contrasting the “fast” debt crisis in Figure 3, with the fast, rollover crises, that may materialize as a consequence of “extreme” beliefs. It is easy to see that the debt threshold under extreme beliefs (conditional on $\rho = E$) is always lower than the threshold at which fast crisis can materialize ($\rho = P$):

Proposition 4. *Strict concavity of the utility function implies $\bar{B}(0)_{pes} > \bar{B}(0)_{EX}$*

Proof. See Appendix C.4. □

Under $\rho = P$, new bonds can be issued at the default-risky rate up to $\bar{B}(1)$ —the government is resilient to rollover crises over a wide range of debt. Under $\rho = E$, instead, default risk translates immediately into a loss of market access. This limits greatly the range of sustainable debt levels.

To sum up, the two types of “fast” crises, one driven by extreme and the other by pessimistic beliefs, are different in one, crucial, respect. When $\rho = P$, “fast” debt crises

occur when investors anticipate that the government is not able and/or willing to adjust enough to keep the stock of debt below $\bar{B}(1)$ at the default-risky rate. When $\rho = E$, instead, “fast” crises occur when investors find that the government will choose to default immediately unless they are willing to roll over sovereign debt.

4 Multiplicity with Long-Term Debt

For any given stock of debt, longer maturities may help sustainability, by reducing the exposure to rollover risk and the pass-through of hikes on interest rates onto the total cost of servicing the outstanding debt. An important question is whether and under what circumstances maturity can rule out multiplicity leading to either slow-moving or to fast debt crises.

In this section, we present a numerical example of Figure 1-3 with long-term debt, calibrating our model with standard parameter values. We keep assuming that beliefs are “static”—that is, agents in the economy do not attribute a positive probability to a switch in the regime of expectations. We relax this assumption in the next section. We focus our analysis on the cases of optimistic and pessimistic beliefs, and briefly comment on extreme beliefs at the end of the section. Overall, we find that multiplicity of equilibria remains pervasive.

4.1 Calibration

In solving the model with long-term debt, we posit the following functional form for the utility function:

$$\mathcal{U}(c, g) = \log(c) + \gamma \log(g - \bar{g}); \quad (13)$$

In our calibration, we set benchmark parameters following [Conesa and Kehoe \(2017\)](#). The parameter values are shown in Table 1.

As shown in the table, we normalize output \bar{y} to 100 so that the units in the model can be interpreted as percentage of GDP: e.g. $B = 50$ means that debt to GDP ratio is 50% in normal times. We set cost of default as $5\% = 1 - Z$. Our default cost is lower relative to the literature (e.g. [Alesina et al. \(1992\)](#)), on the grounds that we assume this cost to be permanent.¹⁵ We assume the relative weight of government utility is 0.2; sensitivity analysis shows that this parameter is unimportant for our result.

The severity of recession A is set to 0.9 so that a recession results in a decrease in output by 10% for the benchmark scenario. This parameter is crucial to generating gambling for

¹⁵Upon a default, in our baseline scenario Z cannot be too small in that the government spending cannot fall below \bar{g} . In other words, the conditions $\tau AZ\bar{y} > \bar{g}$ and $\tau Z\bar{y} > \bar{g}$ must be satisfied.

Table 1: Parameter values, baseline

\bar{y}	Output	100
Z	Cost of Default	0.95
β	Discount factor	0.98
γ	Relative weight of c and g in the utility function	0.20
τ	Government revenue as a share of output	0.36
\bar{g}	Level of the critical government expenditure	25
A	Fraction of output during recession	0.9
p	Probability of leaving the recession	0.2
δ	Amortization rate of market debt	0.2

recovery in an optimistic world. A more severe recession leads to a stronger smoothing motive for the government, which may induce the government to choose a high-debt risky-debt strategy—we report results for different A in Appendix I.

We set the critical government expenditure \bar{g} at 25% of GDP: the higher this value, the smaller the room for discretionary spending. Government revenue as a fraction of output is determined by the constant tax rate τ . In normal times, the government income is 36, but in a recession, it drops to 32. We posit $\delta = 0.2$ to match average maturity from 2000-2009 for Greece, Italy and Spain, which is about 5 years. We set $p = 0.2$ so that the expected waiting time for recovery is 5 years.

4.2 Pervasive Multiplicity

The key novel result from our analysis is that, as the debt tolerance threshold in a recession moves with investors’ expectations, a switch to pessimistic beliefs might result in “fast” debt crises—a result that holds also when debt has long maturity. Figure 4 plots the policy functions conditional on a recession, together with the debt tolerance thresholds (in a recession and in normal times), in the optimistic world (left panel) and the pessimistic world (the right panel).

A striking feature of the optimistic world—on the left panel of Figure 4—is the high value of the debt tolerance threshold in a recession, about 206% of GDP (or 186% as a ratio of GDP in normal times). As further discussed in Section 6, in our exercises we find that $\bar{B}(0)_{opt}$ is generally not sensitive to the probability of recovery p or debt maturity δ . In a recession, the government smooths consumption by borrowing at the risk-free rate until debt level reaches $\bar{B}(0)_{opt}$: the figure suggests that the dynamics of debt are mildly increasing.

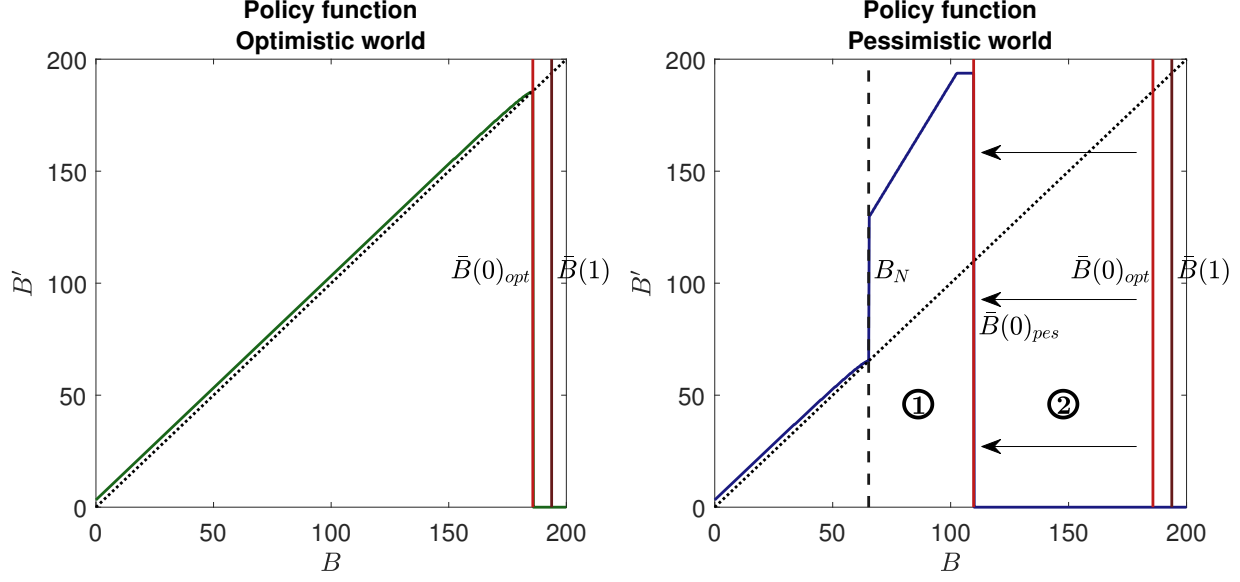


Figure 4: Policy functions in the optimistic world and the pessimistic world

The right panel of Figure 4 depicts a situation in which investors unexpectedly change their view on government solvency, from optimistic to pessimistic. While $\bar{B}(1)$ is not affected (because of our assumption that, after recovering, the economy never falls back into a recession again), the consequences of such a change on the debt tolerance threshold in a recession are stark. There is a large drop from $\bar{B}(0)_{opt}$ to $\bar{B}(0)_{pes}$.

Yet, if the initial debt is in the region between 0 and B_N , the country is barely affected by the switch in the regime of beliefs. The government is still able to borrow at the risk-free rate, and, as a result, it keeps increasing the level of debt for smoothing purposes till the stock reaches B_N . At this level, the government reduces its deficits so to keep the debt stock stationary, waiting for a recovery.

The switch in expectations is instead consequential if the initial debt is anywhere above B_N but below $\bar{B}(0)_{pes}$, i.e., in the region labelled ① in our figure. In this region, the government also keeps borrowing, but at higher interest rates, letting debt accumulate at a faster pace. Investors anticipate that default may then occur depending on whether the economy fails to recover in the next period. This is a scenario of a “slow-moving” crisis, in that default is preceded by debt accumulation. Note that, under our parameterization, a slow-moving crisis can arise for a debt to GDP ratio as low as 65% of GDP in normal times (about 72% of GDP in a recession).

Furthermore, if debt is in the region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ —the region labelled ② in the figure—the crisis is of the type that we dub “fast”: it occurs simultaneously with the shift in beliefs. It should be stressed that in this “fast crisis” region, as long as investors are optimistic, the government can actually issue debt at the risk-free rate. But once investors

change their view and charge high risky rates, they understand that the sovereign will be unwilling to reduce its financing need enough to keep new issuance below $\bar{B}(1)$. The debt market dries out. Facing such a rollover crisis, the government defaults immediately. There is no “slow-moving” accumulation of debt. In our calibration, fast crises can occur with a debt to GDP ratio in a recession between about 122% and 206%.

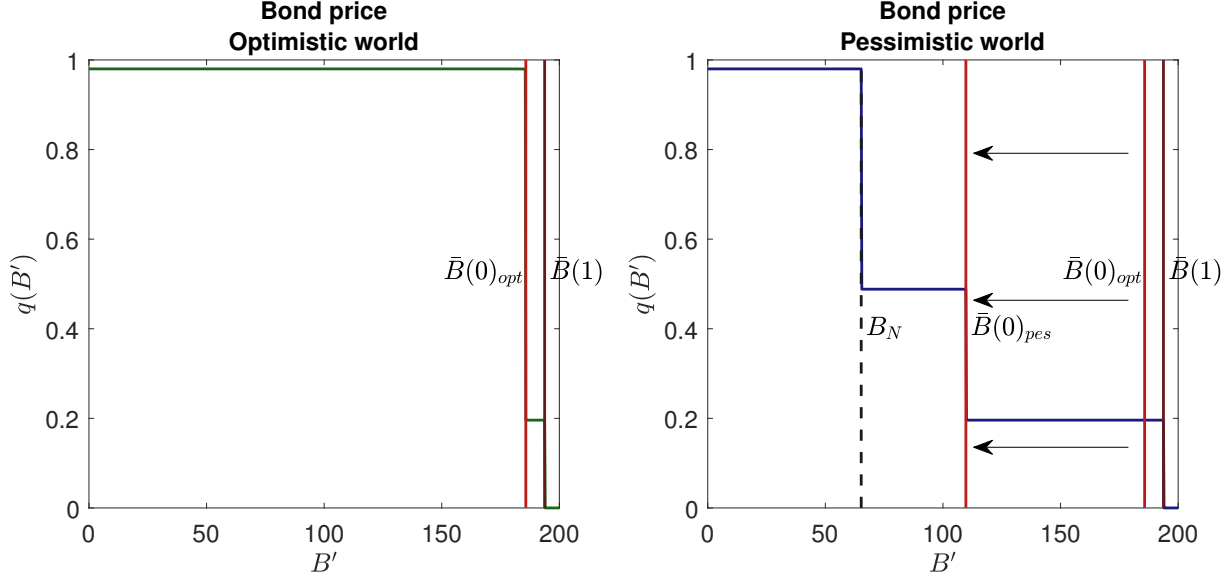


Figure 5: Bond prices in the optimistic and the pessimistic world

In Figure 5, we plot the price of new issuance of government bonds in both the optimistic and the pessimistic worlds, contingent on the economy being in a recession. The left panel in the figure shows that this price remains high for a wide range of debt-to-GDP ratio in the optimistic world. The price drops very markedly in the narrow region between $\bar{B}(0)_{opt}$ and $\bar{B}(1)$. The right panel of Figure 5 illustrates the impact of a change in investors’ expectations, from optimistic to pessimistic. If the amount of new issuance lies between B_N and $\bar{B}(0)_{pes}$, the government is exposed to the possibility of slow-moving debt crises next period—a crisis will occur if the economy remains in a recessionary state. Thus, the bond price drops to 0.48.¹⁶ If new issuance is above $\bar{B}(0)_{pes}$ but below $\bar{B}(1)$, instead, the government is at the risk of “fast” debt crises, which decreases the bond price even further.¹⁷

¹⁶Different from long-term bonds, in Figure 1-3, the price of one-period bonds remains risk-free for all levels of new issuance below $\bar{B}(0)_{pes}$. When the amount of new issuance lies in between B_N and $\bar{B}(0)_{pes}$, the government is exposed to slow-moving crisis in the following periods. However, one-period debt is still risk-free in the sense that default does not occur immediately in slow crisis and government bonds are fully rolled over every period.

¹⁷We leave to the Appendix D a numerical example assuming one-period bonds ($\delta = 1.0$), using the same calibration as in this section. Comparing the result for one-period bonds with Figure 4, shows that, as δ

When we repeat the analysis for the case of extreme beliefs, we find that long maturities have a significant impact on debt sustainability, as they reduce the size of the period-by-period financing need. However, under these beliefs, the debt level at which rollover crises occur is quite low. Under our calibration, the debt tolerance threshold, above which extreme beliefs can cause a rollover crisis is 8 percent when debt is short term ($\delta = 1$), and rises to 38 percent as the maturity lengthens up to our baseline calibration ($\delta = 0.2$).

We conclude by stressing that, while a long debt maturity can substantially increase the debt thresholds in a pessimistic world (improving government welfare), it may not rule out self-fulfilling crises. Threats of both “slow-moving” and “fast” debt crises are still pervasive with long-term debt.

5 Deleveraging and Debt Dilution

So far we have carried out our analysis under the assumption that, when in an optimistic mode, investors and the government attribute zero probability to the bad equilibrium. In this section, we relax this assumption and construct sunspot equilibria in which all agents in the economy anticipate the possibility of a change in beliefs regime, heavily drawing on the approach by [Conesa and Kehoe \(2017\)](#).

We consider two exercises where the beliefs state ρ transits between optimistic and pessimistic ($\rho \in \{O, P\}$), and between optimistic and extreme ($\rho \in \{O, E\}$).¹⁸ Specifically, we assume that investors are initially optimistic on government solvency, but all agents in the economy are aware that market views may turn pessimistic/extreme with probability π . If this pessimistic/extreme view is self-fulfilling, investors stick to pessimistic/extreme beliefs *forever* afterwards. A switch in beliefs occurs if and only if pessimistic/extreme expectations are self-validating. Debt tolerance threshold in a recession is now denoted as $\bar{B}(0)_\pi$. We posit a small sunspot probability, equal to $\pi = 0.04$. To save space, in the following we focus on policy functions and bond price schedules in a recession.

5.1 Optimistic and Pessimistic Beliefs

We first present a sunspot equilibrium where beliefs may transit from optimistic to pessimistic. Pessimistic beliefs are self-fulfilling if the government, when a switch occurs, either

converges to unity, $\bar{B}(0)_{pes}$ and B_N are much lower, while $\bar{B}(0)_{opt}$ is not affected at all. We elaborate on these results in Appendix D and Section 6.

¹⁸When a switch is possible across all types of beliefs ($\rho \in \{O, P, E\}$), the model results lie between $\rho \in \{O, P\}$ and $\rho \in \{O, E\}$, i.e., more deleveraging than $\rho \in \{O, P\}$ but more leveraging than $\rho \in \{O, E\}$. For the sake of expositional clarity, we restrict our attention to equilibria in which a switch is possible only between two regimes.

borrow more than $\bar{B}(0)_{pes}$ (slow-moving crises), or defaults (“fast” crises).

Our key result is that, in the sunspot equilibrium, the government may choose to decrease debt to safe levels in a recession, motivated by large gains in expected utility from either eliminating belief-driven crises altogether—so to avoid the welfare ‘cliff effect’ created by these crises—, and/or lowering borrowing costs—so to take advantage of the ‘price effect’ from reducing financial vulnerability. However, different from [Cole and Kehoe \(2000\)](#), deleveraging is preferred over debt accumulation only for a small range of debt above the threshold at which slow-moving crises become a possibility. For a very wide range of debt levels, the government prefers to accumulate liabilities and smooth consumption, gambling on the prospective recovery. The incentive to deleverage is however stronger, the shorter the maturity of debt.

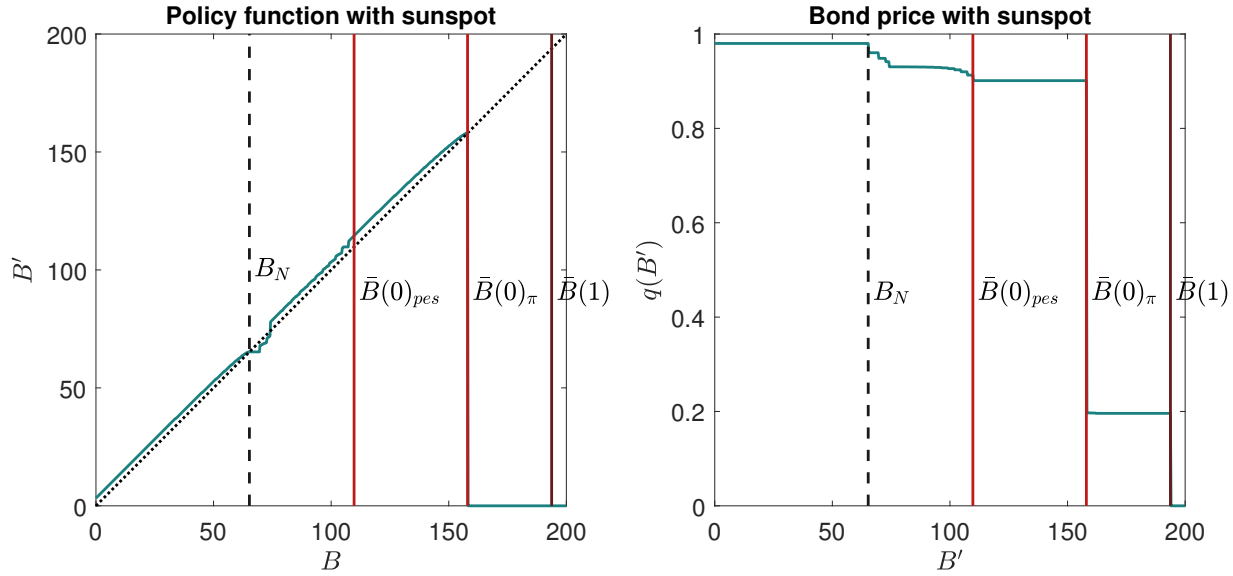


Figure 6: $\delta = 0.2$ and $\rho \in \{O, P\}$

Figure 6 displays the policy function (left) and the bond price function (right) in sunspot equilibrium with long-term debt, where investors assign a small probability to a switch to pessimistic beliefs, hence $\rho \in \{O, P\}$. For debt levels in the region between 0 and B_N , the debt dynamics are the same as in the right panel of Figure 4, and the government is able to issue safe debt.

In the region between B_N and $\bar{B}(0)_\pi$, where the economy is vulnerable to sunspot crises, the debt dynamics are different from what we have seen so far—it is no longer uniform. This region can indeed be split into two subregions. For an initial debt level close to B_N , the government chooses to run surpluses and reduce its borrowing. This allows the government to avoid high and increasing borrowing costs, as well as a large utility loss, if self-fulfilling pessimistic expectations materialize. However, for large enough initial debt, the government

prefers to keep borrowing. It will do so until its debt level reaches $\bar{B}(0)_\pi$, even for debt levels above (but close to) $\bar{B}(0)_{pes}$, where self-fulfilling crises, if they occur, are “fast”.¹⁹

Why is deleveraging optimal for debt levels just above B_N , but not so for debt levels just above $\bar{B}(0)_{pes}$? The key insight is that keeping debt below $\bar{B}(0)_{pes}$ shields the country from “fast” crises, but not from slow-moving ones. Hence, while the government may still have some advantage not to let debt trespass $\bar{B}(0)_{pes}$, this advantage is exclusively in terms of lower borrowing costs (as shown on the right panel of Figure 6), not in terms of eliminating the possibility of crises ‘tout-court’ (the welfare ‘cliff effect’ is less relevant here).²⁰ The borrowing costs advantage (about 1.4%)²¹ is not enough to offset the benefits from smoothing consumption in a recession via borrowing. Crucial to this result is the low pass-through of higher interest rates into borrowing costs when the outstanding stock debt has long maturity—an observation that resonates with the analysis of ‘debt dilution’ in Aguiar and Amador (2020).²² The gains in terms of lower borrowing cost from deleveraging are shared between the investors (as a capital gain) and the government (the ‘price effect’). As the maturity is long, the former component, which does not provide any incentive to deleverage, accounts for a larger share and therefore weakens the ‘price effect’. This is not the case for short-term debt—absent ‘debt dilution’, the benefits from lower borrowing costs are a much stronger incentive to deleverage.

For comparison, we show the case of one-period bonds, with $\delta = 1.0$, in Figure 7. Relative to the long-term maturity case, three differences are apparent. First, levels and shifts in thresholds are now substantially different from Figure 6. Second, the bond price remains risk-free in the region between B_N and $\bar{B}(0)_{pes}$, in which a switch in expectations may end up igniting a slow-moving debt crisis. As debt is fully rolled over each period, a risk of slow-moving crises, which may eventually lead to default, does not have any impact on (short-term) bond prices.²³

Last but not least, the ‘price’ and ‘welfare cliff’ effects of belief-driven crises play a somewhat different role in shaping government decisions when government debt is short-term. As for the case of long-term debt, deleveraging is optimal for a limited range of debt above B_N . But now deleveraging is also optimal for a very small range of debt just above $\bar{B}(0)_{pes}$. The right panel of the graph confirms the insight already discussed above. There

¹⁹We find that a government exposed to the risk of fast rollover crises ($B > \bar{B}(0)_{pes}$) with long-term bonds accumulates liabilities faster. This is shown in Appendix E.

²⁰Discontinuity in value function, like a ‘cliff’ in a pessimistic world, motivates the government to deleverage. See Appendix F for details.

²¹This is obtained by $\frac{1-q(\bar{B}(0)_{pes}+1)}{q(\bar{B}(0)_{pes}+1)} - \frac{1-q(\bar{B}(0)_{pes})}{q(\bar{B}(0)_{pes})}$. We use the same formula to derive yield difference of one-period bonds in our next simulation, which is 3.4%.

²²See also Aguiar et al. (2019).

²³In this sense, bond prices alone may not contain enough information to infer sunspot probability/market beliefs—a beliefs state ρ pins down the equilibrium bond price, but not vice-versa.

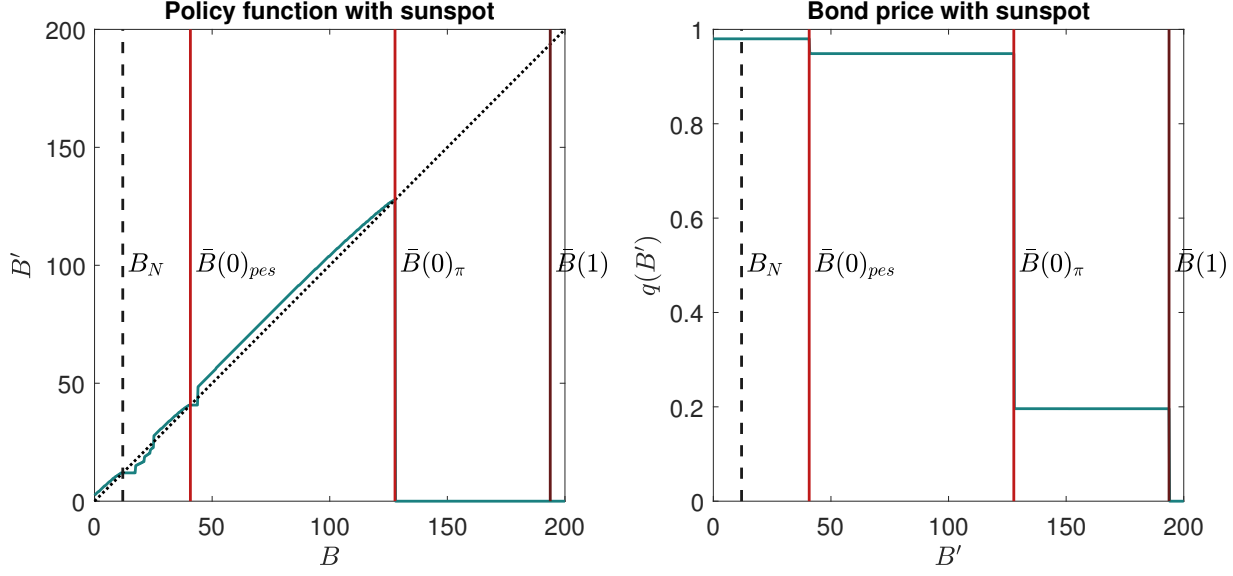


Figure 7: $\delta = 1.0$ and $\rho \in \{O, P\}$

is no price effect in trespassing the threshold B_N . The government deleveraging decision however reflects the prospective loss of welfare, which is substantial because of the ‘cliff effect’.

Conversely, although the cliff effect is absent at the threshold $\bar{B}(0)_{pes}$, the price effect at this threshold is now much higher. There is full pass-through of changes in market interest rates (about 3.4%) on government borrowing costs just in one period. That is to say, at short maturities, the government internalizes the gains from reducing the borrowing cost fully, while longer maturity does not provide enough incentive (as the gains are shared) to save for debt levels close to $\bar{B}(0)_{pes}$ (see again [Aguiar and Amador \(2020\)](#)).

5.2 Optimistic and Extreme Beliefs

We now turn to the case of a sunspot where agents attach a small positive probability to a switch from optimistic to extreme beliefs, i.e., $\rho \in \{O, E\}$.²⁴ Relative to the previous subsection, below we show that, when $\rho \in \{O, E\}$, the government deleverages over a much wider range of debt, and yield rates on long-maturity debt are much higher—with bond prices responding more strongly to the sunspot probability.

²⁴Extreme beliefs are self-fulfilling, if the government, facing a rollover crisis, defaults immediately. Bearing in mind that the utility off the equilibrium path in [Conesa and Kehoe \(2017\)](#) is the same as (3), policy functions and bond price schedules in this type of sunspot equilibria are consistent with [Conesa and Kehoe \(2017\)](#). The main difference is that we set $\kappa = 1 - \beta + \beta\delta$ in the budget constraint $g + \kappa B = \tau y(s) + q(B', s)(B' - (1 - \delta)B)$ so that the default-free bond price is β for all debt maturities. In contrast, the price of default-free long-term bonds is smaller than β in [Conesa and Kehoe \(2017\)](#) as κ is smaller than $1 - \beta + \beta\delta$.

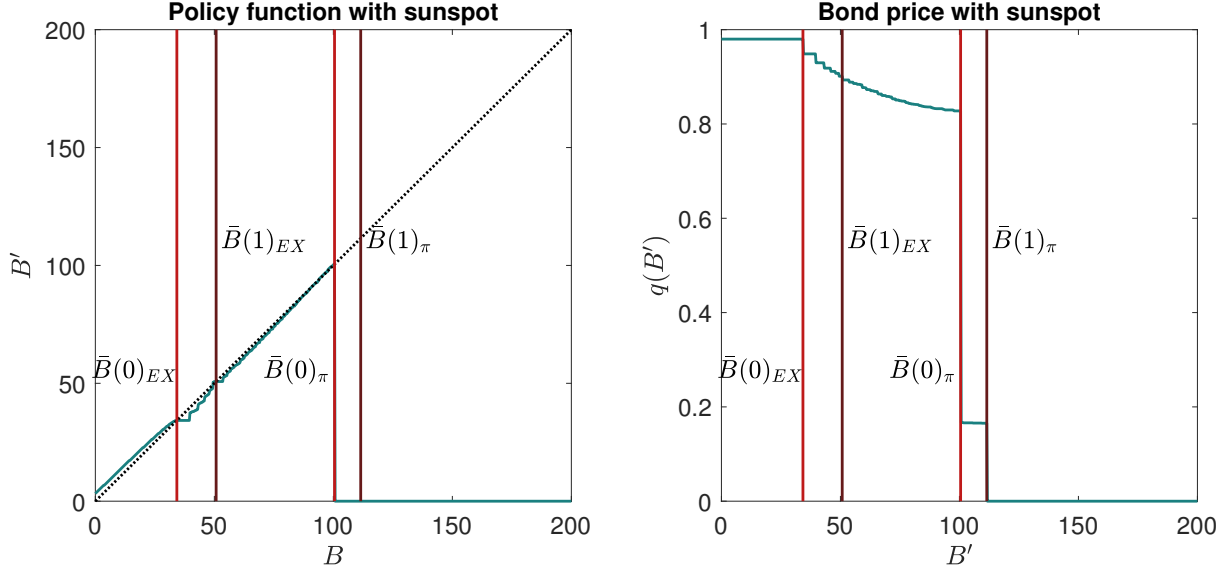


Figure 8: $\delta = 0.2$ and $\rho \in \{O, E\}$

Figure 8 shows the policy function and the bond price schedule for our calibration with long-term bonds. To study the sunspot equilibrium with extreme beliefs, we define a new debt threshold conditional on recovery, denoted $\bar{B}(1)_{\pi}$. The reason to introduce this new threshold is that, with extreme beliefs, the government remains exposed to the risk of rollover crises even after the economy exits the recession, if its outstanding debt level is large enough—indeed larger than $\bar{B}(1)_{EX}$.²⁵ This added vulnerability deteriorates the utility of repaying conditional on the recovery, hence $\bar{B}(1)_{\pi}$ in the figure is lower than $\bar{B}(1)$ in Figure 6.

As shown in the figure, when debt is in the region $[0, \bar{B}(0)_{EX}]$, the government borrows in a recession, until outstanding debt reaches $\bar{B}(0)_{EX}$. With debt in the region $(\bar{B}(0)_{EX}, \bar{B}(0)_{\pi}]$, however, the debt dynamic is quite complex. The government mostly prefers to deleverage (running surpluses) unless debt is lower than but close to either $\bar{B}(1)_{EX}$ or $\bar{B}(0)_{\pi}$. Deleveraging is optimal over more than 80% of this debt region. In contrast, the government borrows (gambling on a recovery) only over 10% of the region; in 5% of this debt region, the government prefers to keep the debt level unchanged.

The right panel of Figure 8 displays the bond price, which is clearly lower relative to Figure 6. For instance, when B' is 70 (77.8% of GDP), the bond yield surges to 16.5%, relative to 5.4% in Figure 6. Note that the yield raises smoothly as debt rises from $\bar{B}(0)_{EX}$ to $\bar{B}(0)_{\pi}$ —with the pace slowing down as B' approaches $\bar{B}(0)_{\pi}$. Intuitively, as the level of outstanding debt B approaches $\bar{B}(0)_{\pi}$, the bond price does not deteriorate enough to provide

²⁵Even after the recovery, a switch to extreme beliefs may lead to default—these beliefs are self-validating when a collapse of bond market induces an immediate default, regardless of the output states. Derivation of $\bar{B}(1)_{EX}$ is shown in footnote 12.

a significant incentive to deleverage. The opposite is true, however, if the initial debt level is low enough (far enough from $\bar{B}(0)_\pi$): bond prices drive government decisions to deleverage. The incentive to run surpluses however is weaker if the outstanding debt is low enough and close to $\bar{B}(1)_{EX}$.

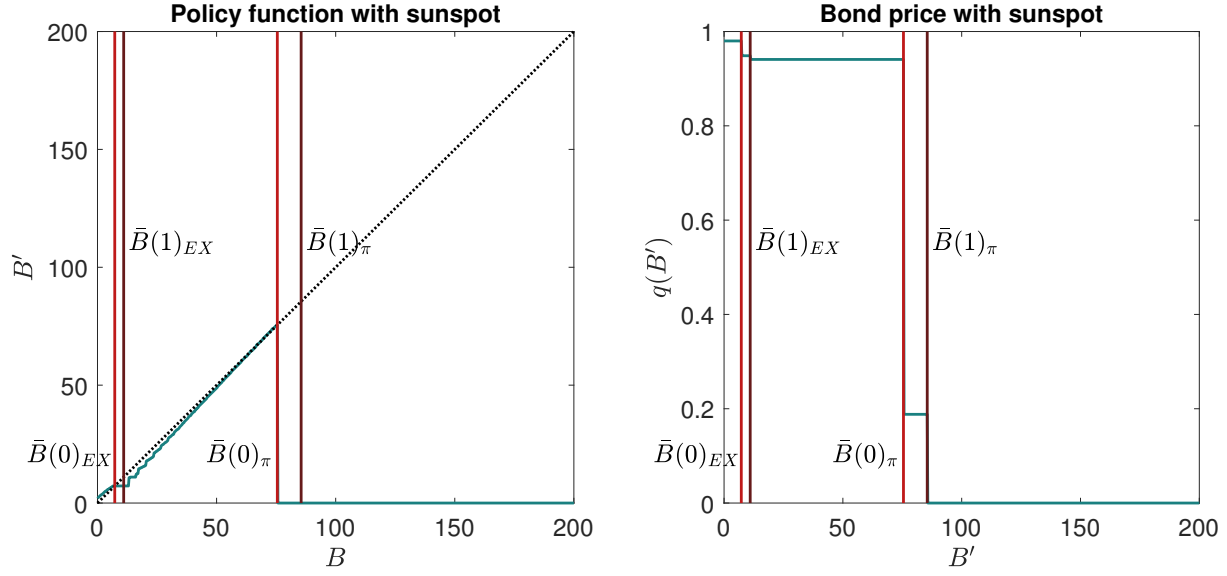


Figure 9: $\delta = 1$ and $\rho \in \{O, E\}$

For comparison, Figure 9 illustrates the sunspot equilibrium with short-term bonds. With short-term debt, debt thresholds are much lower, and the deleveraging region is wider. Deleveraging is indeed optimal over more than 85% of the region between $\bar{B}(0)_{EX}$ and $\bar{B}(0)_\pi$; running deficits remains optimal over about 10% of the region. Investors systematically price the rollover crisis risk, with sharp adjustment across each threshold. The price incentive substantially strengthens the incentive to adopt prudent policies and keep debt on a declining path.

6 Resilience to Self-Fulfilling Crises

In this subsection we study how an economy may/may not be vulnerable to debt crises, depending on the maturity of its debt and the depth and expected persistence of a downturn. To do so, we carry out sensitivity analysis comparing our baseline with the debt-limit framework, focusing on the cases of optimistic and pessimistic beliefs.²⁶ We highlight conditions

²⁶For the sake of space, we do not include the case of extreme beliefs in this section. We note that, under these beliefs, a longer maturity and/or a higher probability of recovery have the strongest effect on the threshold $\bar{B}(0)_{EX}$ conditional on a recession. Changing these parameters, everything else equal, raises this threshold much more compared to the shift in $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$.

under which fast debt crises can be ruled out altogether.

For our baseline model, in Figure 10 we plot debt tolerance thresholds in a recession against variable debt maturity (left panel) and probability of recovery (right panel).

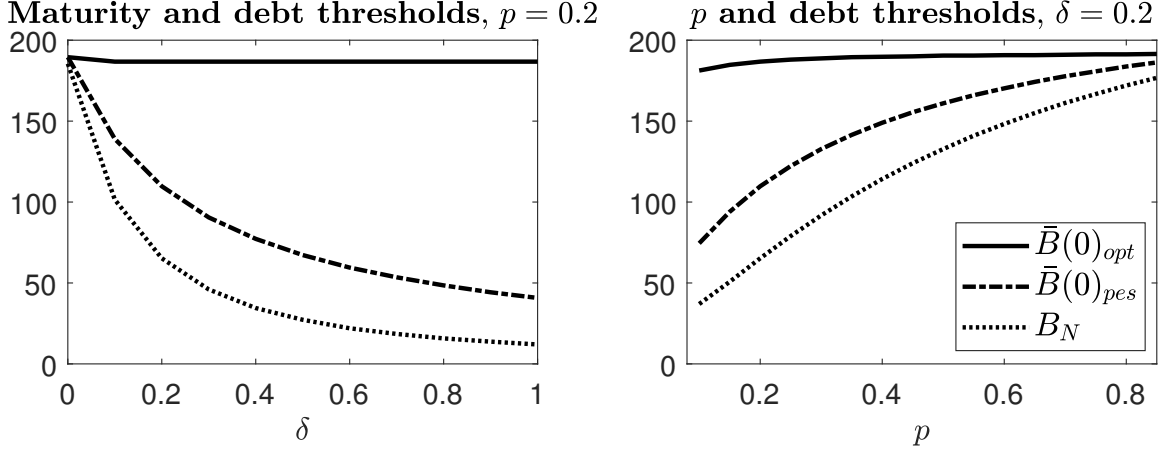


Figure 10: Debt thresholds in the baseline model given $A = 0.9$

Starting from the left panel of Figure 10, we first note that $\bar{B}(0)_{opt}$ is largely insensitive to debt maturity, but for extremely small values of δ ($\delta \rightarrow 0$), corresponding to very long maturities. When investors are optimistic, as long as the government keeps debt below $\bar{B}(0)_{opt}$, so that it can borrow at the risk-free rate, long-term and short-term debt are basically equivalent. When $\delta \rightarrow 0$, however, $\bar{B}(0)_{opt}$ increases slightly. Intuitively, consider the limit case of “consols” (perpetuities), where there is no need to roll over the outstanding debt stock. Even if the interest rate on new issuance is high—raising borrowing costs at the margin—, high rates deteriorate the value of outstanding bonds harming investors—the government benefits from ‘diluting’ existing debt. When the amount of debt inherited is large enough, the benefit from debt dilution outweighs the higher borrowing costs: a government has an incentive to issue debt above the threshold $\bar{B}(0)_{opt}$. A high-debt high-risk issuance strategy yields higher benefit/utility relative to a low-debt low-risk debt issuance strategy, i.e., $V_{O,1}^R < V_{O,2}^R$ in (4) and (5). For consols, in line with Proposition 3, $\bar{B}(0)_{opt}$ equals to $\bar{B}(0)_{pes}$.

In contrast, both B_N and $\bar{B}(0)_{pes}$ decrease sharply with δ , that is, they increase with a longer maturity of debt. To see why, consider the net bond revenue in a pessimistic world, $\beta p(B' - (1 - \delta)B) - \kappa B$, where $\beta p B'$, $\beta p(1 - \delta)B$ and κB denote, respectively, revenue from newly issued bonds, the value of the outstanding stock of bonds, and interest payment to investors. Maturity has opposite effects on net bond revenue. As the maturity of bonds becomes longer ($\delta \downarrow$), the value of the outstanding stock of bonds $\beta p(1 - \delta)B$ rises but the interest payments due in the period κB fall. The first effect decreases, while the second effect increases the net bond revenue. Rearranging the net revenue equation as follows:

$\beta p B' - [1 - \beta(1 - p)(1 - \delta)]B$, it is apparent that the second effect always dominates the first one: a fall in δ unambiguously increases net debt revenue—explaining why $\bar{B}(0)_{pes}$ and B_N are larger as the debt maturity becomes longer.²⁷

Observe that, in the panel, the “fast” crisis zone—the distance between $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$ —becomes wider as the debt maturity becomes shorter. On the contrary, the “slow-moving” crisis zone—the distance between B_N and $\bar{B}(0)_{pes}$ —remains approximately unchanged as maturity shortens. B_N and $\bar{B}(0)_{pes}$ basically fall at a similar pace.

To gauge whether longer debt maturities can eliminate multiplicity, let $\delta \rightarrow 0$ in the left-hand side panel. As debt maturity grows long enough, $\bar{B}(0)_{opt}$ coincides with $\bar{B}(0)_{pes}$: “fast” crises no longer exist. However, slow-moving crises are still possible, as B_N never coincides with $\bar{B}(0)_{opt}$. Not even “consols” can rule out the multiplicity that leads to slow-moving crises—these remain pervasive for all debt maturities.

The right panel of Figure 10 shows that the probability of recovery p does not have much of an effect on $\bar{B}(0)_{opt}$, while it has a significant impact on both B_N and $\bar{B}(0)_{pes}$. The net bond revenue in an optimistic world, $\beta(B' - (1 - \delta)B) - \kappa B$, does not vary with p , while the net bond revenue in a pessimistic world, $\beta p(B' - (1 - \delta)B) - \kappa B$, is unambiguously increasing in p . A higher probability of recovery p significantly narrows the “fast” crisis zone. It also narrows, but to a lesser extent, the “slow-moving” crisis zone.

We find it instructive to extend our sensitivity analysis and encompass a framework widely discussed in the literature under the headline of “debt-limit” (as opposed to “strategic default”) model, see [Lorenzoni and Werning \(2019\)](#) for a leading recent contribution. In this framework, the government is assumed to be willing to exhaust all possibilities of adjustment before repudiating the debt and thus fall in the costly low-output financial-autarky state. The default condition can then be written as follows:

$$\mathcal{PS}^{Max} + \max_{B'} \{q(B', s)(B' - (1 - \delta)B)\} < \kappa B \quad (14)$$

where \mathcal{PS}^{Max} is the period-by-period maximum adjustment in the primary surplus. A detailed analysis is given in Appendix G, where we derive the debt-limit model as a variant of our baseline.²⁸ Hereafter, we report the results of our sensitivity analysis using the debt-limit variant of our model, relative to debt maturity and probability of a recovery. Results are shown in Figure 11.

Firstly, looking at the left panel in Figure 11, it is apparent that long debt maturities do not rule out multiplicity. In the debt-limit framework, slow-moving debt crises remain

²⁷One could note that, when investors hold a pessimistic view of the government, an official swap of short-term bonds for long-term bonds may improve the debt tolerance threshold of a country (a point discussed is detailed in [Corsetti et al. \(2017\)](#)).

²⁸The analyses of the debt-limit model in the appendix and the sensitivity analysis in the text come into handy in comparing our model with [Lorenzoni and Werning \(2019\)](#).

pervasive. Secondly, the debt threshold $\bar{B}(0)_{opt}$ is insensitive to debt maturity only for a δ higher than 0.57, that is, for relatively short maturities. For longer maturities, different from the left panel of Figure 10, $\bar{B}(0)_{opt}$ becomes the same as $\bar{B}(0)_{pes}$ —both increasing as δ falls. Recall that, by the logic of the Laffer curve, issuing less debt at the risk-free rate may yield more revenues than issuing more debt at the risky price. In our specification with optimistic expectations, this is the case when debt maturity is sufficiently short. The opposite is true, however, once δ becomes smaller than 0.57, i.e., for long debt maturities.²⁹ The remarkable implication is that long maturities may allow the government to borrow more and smooth adjustment, up to avoiding “fast” crises—corresponding to the range of debt where $\bar{B}(0)_{opt}$ coincides with $\bar{B}(0)_{pes}$. The conditions for this outcome are weaker in the debt-limit framework than in our baseline.

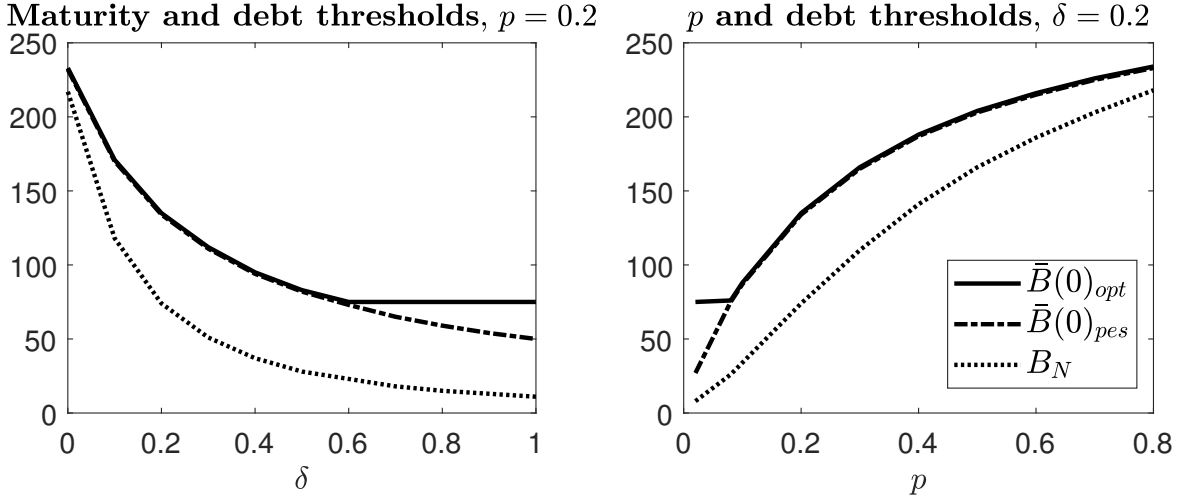


Figure 11: Debt thresholds in the debt-limit framework given $A = 0.9$

A similar picture emerges from the right panel of Figure 11, which plots the debt thresholds against the probability of recovery p . Different from the corresponding panel in Figure 10, $\bar{B}(0)_{opt}$ now rises substantially with a higher p , and coincides with $\bar{B}(0)_{pes}$ for any p larger than 0.08. In an optimistic world, only for a very low probability of recovery, the low-debt safe-debt issuance strategy yields more revenue than the high-debt risky-debt issuance strategy, causing $\bar{B}(0)_{opt}$ to diverge from $\bar{B}(0)_{pes}$, and to remain insensitive to p . For any non-negligible probability of recovery (p larger than 0.08), the risky-debt strategy generates higher revenue and the “fast” crises zone disappears.³⁰

Overall, our sensitivity analysis shows that, different from [Lorenzoni and Werning \(2019\)](#),

²⁹We characterize this finding also in Appendix G.3.

³⁰The parameter restrictions that rule out “fast” debt crises in baseline are much more stringent. That is, the country is in a deep recession, the probability of recovery is high, and debt maturity is sufficiently long. These results are shown in Appendix I.

multiplicity remains pervasive for *all debt maturities* both in our baseline and debt-limit framework—fast (rollover) crises may be eliminated while the risk of slow-moving ones is prevalent.³¹ For intermediate debt maturities and non-negligible probability of recovery, rollover crises are still possible in our baseline, but not in the debt-limit framework. The reason is that, in the latter, the government takes advantage of long debt maturities and good recovery prospect to raise revenue by issuing (more) risky debt—de facto putting the economy in a slow-moving debt crisis mode.

7 Conclusion

The literature has long emphasized that, once a country debt is sufficiently high, the equilibrium is no longer unique and the country is vulnerable to disruptive self-fulfilling crises. As the COVID-19 pandemic is causing widespread economic crises across the globe, it is unavoidable that debt stocks rise virtually everywhere, potentially undermining stability in the bond markets in advanced countries and raising issues in which instruments are available to keep these markets in a “good equilibrium”.

This paper shows that different types of self-fulfilling crises, one emphasized by [Calvo \(1988\)](#), the other by [Cole and Kehoe \(2000\)](#), may occur in the same dynamic [Calvo \(1988\)](#) setting. In particular, both slow-moving debt crises and rollover crises are possible when investors coordinate on what we dub “pessimistic” beliefs, while rollover crises are a form of self-fulfilling debt crises specific to “extreme” beliefs.

We revisit debt dynamics and the incentive to deleverage when governments operate under the threat of self-fulfilling debt crisis. This is an important issue, that may dominate debates on fiscal policy in the post-COVID, high-debt regime. Under the threat of rollover crises driven by investors coordinating on “extreme” beliefs, in line with the literature, we show that a forward-looking benevolent government finds it optimal to reduce debt even during recessions. As a contribution to the literature, however, we also show that, if crises are anticipated to be slow-moving—driven by “pessimistic” beliefs—, deleveraging is optimal only over a relatively small range of debt. This result suggests that debt may remain at high levels for a long time, even if governments are aware that their failure to deleverage keeps their country exposed to the threat of belief-driven crises. We stress that this result is obtained independently of political economy considerations, with policymakers modelled as

³¹Differences in our results relative to [Lorenzoni and Werning \(2019\)](#) originate from a different assumption concerning output uncertainty. In [Lorenzoni and Werning \(2019\)](#), output is drawn from normal distribution with a single peak—which in their debt-limit framework rules out multiple equilibria for one-period bonds. Our model, instead, features a bimodal distribution for which, as explained in footnote 6, stable equilibria exist at high interest rates for all debt maturities.

short-sighted or self-interested. Even for forward-looking benevolent governments, the threat of slow-moving debt crises is generally not enough to motivate precautionary fiscal policy of risk reduction. In light of cross-border contagion effects undermining stability at global level, there is an argument for international fiscal compacts associated with institutional liquidity provision.

As a direction for future research, an extension of our model may shed light on debt sustainability when the government can rely on external bailouts and/or liquidity assistance. The logic of our model suggests that the effect of bailouts on the dynamics of debt and vulnerability to crisis would depend on the regime of beliefs, in turn affecting the incentive of the government to gamble on prospective recovery. In some cases, bailouts may help and speed up deleveraging. In other cases, they may give an extra incentive to smooth adjustment by borrowing. Bailouts may thus create a trade-off between resilience to rollover crises and vulnerability to default at high levels of debt. Understanding these trade-offs is crucial in the design of an efficient governance of official lending institutions.

References

- Aguiar, M. and M. Amador (2014). Sovereign Debt. In *Handbook of International Economics*, Volume 4, Chapter 11, pp. 647–687. Elsevier B.V.
- Aguiar, M. and M. Amador (2020). Self-Fulfilling Debt Dilution: Maturity and Multiplicity in Debt Models. *American Economic Review* 110(9), 2783–2818.
- Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2013). Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises. *NBER Working Paper No. 19516*.
- Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2015). Coordination and Crisis in Monetary Unions. *Quarterly Journal of Economics* 130(4), 1727–1779.
- Aguiar, M., M. Amador, H. Hopenhayn, and I. Werning (2019). Take the Short Route: Equilibrium Default and Debt Maturity. *Econometrica* 87(2), 423–462.
- Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016). Quantitative Models of Sovereign Debt Crises. In *Handbook of Macroeconomics*, Chapter 21, pp. 1697–1755. Elsevier B.V.
- Aguiar, M., S. Chatterjee, H. L. Cole, and Z. Stangebye (2020). Self-Fulfilling Debt Crises, Revisited. *PIER Working Paper 20-003*.
- Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics* 69, 64–83.
- Alesina, A., M. D. Broeck, A. Prati, G. Tabellini, M. Obstfeld, and S. Rebelo (1992). Default Risk on Government Debt in OECD Countries. *Economic Policy*, 428–463.
- Alesina, A., A. Prati, and G. Tabellini (1989). Public confidence and debt management: a model and a case study of Italy. *NBER Working Paper No. 3135*.
- Araujo, A., M. Leon, and R. Santos (2013). Welfare Analysis of Currency Regimes with Defaultable Debts. *Journal of International Economics* 89, 143–153.
- Arellano, C. (2008). Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review* 98(3), 690–712.
- Arellano, C. and A. Ramanarayanan (2012). Default and the Maturity Structure in Sovereign Bonds. *Journal of Political Economy* 120(2).
- Auclert, A. and M. Rognlie (2016). Unique equilibrium in the Eaton-Gersovitz model of sovereign debt. *Journal of Monetary Economics* 84, 134–146.

- Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2018). Sovereign Default: The Role of Expectations. *Journal of Economic Theory* 175, 803–812.
- Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2019). Self-Fulfilling Debt Crises with Long Stagnations. *unpublished manuscript*.
- Bacchetta, P., E. Perazzi, and E. V. Wincoop (2018). Self-fulfilling debt crises: What can monetary policy do ? *Journal of International Economics* 110, 119–134.
- Bianchi, J., J. C. Hatchondo, and L. Martinez (2018). International Reserves and Rollover Risk. *American Economic Review* 108(9), 2629–2670.
- Bocola, L., G. Bornstein, and A. Dovis (2019). Quantitative Sovereign Default Models and the European Debt Crisis. *Journal of International Economics* 118, 20–30.
- Bocola, L. and A. Dovis (2019). Self-Fulfilling Debt Crises: A Quantitative Analysis. *American Economic Review* 109(12), 4343–4377.
- Calvo, G. A. (1988). Servicing the Public Debt: The Role of Expectations. *American Economic Review* 78(4), 647–661.
- Chamon, M. (2007). Can Debt Crises be Self-fulfilling? *Journal of Development Economics* 82(1), 234–244.
- Chatterjee, S., D. Corbae, K. Dempsey, and J. Rios-Rull (2016). A Theory of Credit Scoring and the Competitive Pricing of Default Risk. *Working Paper*.
- Chatterjee, S. and B. Eyigungor (2012). Maturity, Indebtedness and Default Risk. *American Economic Review* 102(6), 2674–2699.
- Chatterjee, S. and B. Eyigungor (2019). Endogenous Political Turnover and Fluctuations in Sovereign Default Risk. *Journal of International Economics* 117, 37–50.
- Cole, H. L. and T. J. Kehoe (1996). A Self-Fulfilling Model of Mexico’s 1994–1995 Debt Crisis. *Journal of International Economics* 41(3-4), 309–330.
- Cole, H. L. and T. J. Kehoe (2000). Self-Fulfilling Debt Crises. *The Review of Economic Studies* 67(1), 91–116.
- Collard, F., M. Habib, and J. C. Rochet (2015). Sovereign Debt Sustainability in Advanced Economies. *Journal of the European Economic Association* 13(3), 381–420.
- Conesa, J. C. and T. J. Kehoe (2014). Is it too late to Bail Out the Troubled Countries in the Eurozone? *American Economic Review: Papers & Proceedings* 104(5), 88–93.

- Conesa, J. C. and T. J. Kehoe (2017). Gambling for Redemption and Self-Fulfilling Debt Crises. *Economic Theory* 64(4), 707–740.
- Corsetti, G. and L. Dedola (2016). The Mystery of the Printing Press: Monetary Policy and Self-Fulfilling Debt Crises. *Journal of the European Economic Association* 14(6), 1329–1371.
- Corsetti, G., A. Erce, and T. Uy (2017). Debt Sustainability and the Terms of Official Support. *ADEMU Working Paper*.
- Corsetti, G., A. Erce, and T. Uy (2020). Official Sector Lending Strategies During the Euro Area Crisis. *The Review of International Organisations* 15, 667–705.
- Cruces, J. J. and C. Trebesch (2013). Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics* 5(3), 85–117.
- Eaton, J. and M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies* 48(2), 289–309.
- Galli, C. (2019). Self-Fulfilling Debt Crises, Fiscal Policy and Investment. *unpublished manuscript*.
- Ghosh, A. R., J. I. Kim, E. G. Mendoza, J. D. Ostry, and M. S. Qureshi (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *Economic Journal* 123(566), 4–30.
- Giavazzi, F. and M. Pagano (1989). Confidence Crises and Public Debt Management. In *Public Debt Management: Theory and History*.
- Grauwe, P. D. (2011). The European Central Bank : Lender of Last Resort in the Government Bond Markets? *CESifo Working Paper Series* 3569.
- Hatchondo, J. C. and L. Martinez (2009). Long-Duration Bonds and Sovereign Defaults. *Journal of International Economics* 79(1), 117–125.
- Lizarazo, S. V. (2013). Default Risk and Risk Averse International Investors. *Journal of International Economics* 89(2), 317–330.
- Lorenzoni, G. and I. Werning (2019). Slow Moving Debt Crises. *American Economic Review* 109(9), 3229–3263.
- Marin, E. (2017). Self-fulfilling Crises and Country Solidarity. *unpublished manuscript*.

- Mendoza, E. G. and V. Z. Yue (2012). A General Equilibrium Model of Sovereign Default and Business Cycles. *Quarterly Journal of Economics* 127(2), 889–946.
- Paluszynski, R. (2019). Learning About Debt Crisis. *unpublished manuscript*.
- Roch, F. and H. Uhlig (2018). The Dynamics of Sovereign Debt Crises and Bailouts. *Journal of International Economics* 114, 1–13.
- Stangebye, Z. (2020). Belief Shocks and Long-Maturity Sovereign Debt. *Journal of International Economics* 127.
- Tirole, J. (2015). Country Solidarity in Sovereign Crises. *American Economic Review* 105(8), 2333–2363.
- Yue, V. Z. (2010). Sovereign Default and Debt Renegotiation. *Journal of International Economics* 80(2), 176–187.

A Derivation of $V_P^R(B, 0)$

Conditional on the default-risky bond price βp , the utility of repaying debt $V_P^R(B, 0)$ is characterized by $V_P^R(B, 0) = \max\{V_{P,1}^R(B, 0), V_{P,2}^R(B, 0)\}$:

$$\begin{aligned} V_{P,1}^R(B, 0) = & \max_{0 \leq B' \leq \bar{B}(0)_{pes}} \mathcal{U}(c, g) + \beta \left(pV(B', 1, \rho = P) + (1 - p)V(B', 0, \rho = P) \right) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1 - \tau)A\bar{y} \end{aligned} \tag{15}$$

$$\begin{aligned} V_{P,2}^R(B, 0) = & \max_{\bar{B}(0)_{pes} < B' \leq \bar{B}(1)} \mathcal{U}(c, g) + \beta \left(pV(B', 1, \rho = P) + (1 - p)V^D(0) \right) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1 - \tau)A\bar{y} \end{aligned} \tag{16}$$

Different from (4) and (5), under the pessimistic beliefs regime, the government faces a bond price that is default-risky, i.e., βp . Depending on initial debt level, pessimistic beliefs may/may not be self-validating (see the graphical analysis in Section 3.3).

B The algorithm for computing value functions

B.1 Baseline

The algorithm computes two debt thresholds in an optimistic world, and three debt thresholds in a pessimistic world.

B.1.1 Optimistic beliefs

1. Compute the debt tolerance threshold in normal times $\bar{B}(1)$ by solving:

$$\frac{\mathcal{U}((1 - \tau)\bar{y}, \tau\bar{y} - (1 - \beta)\bar{B}(1))}{1 - \beta} = V^D(1)$$

After the economy recovers, the government optimization problem is deterministic. Thus, the value function in normal times can be characterized by

$$V(B, a = 1) = \begin{cases} \frac{\mathcal{U}((1 - \tau)\bar{y}, \tau\bar{y} - (1 - \beta)B)}{1 - \beta} & \text{if } 0 \leq B \leq \bar{B}(1) \\ V^D(1) & \text{if } \bar{B}(1) < B \end{cases}$$

2. Guess initial values for the threshold $\bar{B}(0)_{opt}$ and the bond price function $\tilde{q}_{opt}(B', 0)$ in a recession.

3. Given the bond price function $\tilde{q}_{opt}(B', 0)$ and $\bar{B}(0)_{opt}$, guess the value function $\tilde{V}(B, 0)$ in an optimistic world. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B |V(B, 0) - \tilde{V}(B, 0)| < \epsilon$.

$V(B, 0) = \max\{V_{O,1}^R(B, 0), V_{O,2}^R(B, 0), V^D(0)\}$, where

$$\begin{aligned}
V_{O,1}^R(B, 0) &= \max_{0 \leq B' \leq \bar{B}(0)_{opt}} \mathcal{U}(c, g) + \beta \left(pV(B', 1) + (1-p)\tilde{V}(B', 0) \right) \\
&\text{s.t.} \quad g + \kappa B = \tau A\bar{y} + \tilde{q}_{opt}(B', 0)(B' - (1-\delta)B), \\
&\quad c = (1-\tau)A\bar{y} \\
V_{O,2}^R(B, 0) &= \max_{\bar{B}(0)_{opt} < B' \leq \bar{B}(1)} \mathcal{U}(c, g) + \beta \left(pV(B', 1) + (1-p)V^D(0) \right) \\
&\text{s.t.} \quad g + \kappa B = \tau A\bar{y} + \beta p(B' - (1-\delta)B), \\
&\quad c = (1-\tau)A\bar{y}
\end{aligned}$$

4. Derive a new value of $\bar{B}(0)_{opt}$ by solving:

$$V(\bar{B}(0)_{opt,new}, 0) = V^D(0)$$

5. Update the bond price function and compute the error. New values of $q_{opt}(B', 0)$ are

$$q_{opt}(B', 0) = \begin{cases} \beta \left(p + (1-p)(\kappa + (1-\delta)\tilde{q}_{opt}(B'(B', 0), 0)) \right) & \text{if } 0 \leq B' \leq \bar{B}(0)_{opt} \\ \beta p & \text{if } \bar{B}(0)_{opt} < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

6. If $\max_{B'} |\tilde{q}_{opt}(B', 0) - q_{opt}(B', 0)| > \epsilon$ or/and $|\bar{B}(0)_{opt} - \bar{B}(0)_{opt,new}| > \epsilon$, set $\tilde{q}_{opt}(B', 0) = q_{opt}(B', 0)$ and $\bar{B}(0)_{opt} = \bar{B}(0)_{opt,new}$, and go back to 3. Else, start to solve the equilibrium problem in a pessimistic world.

B.1.2 Pessimistic beliefs

1. Repeat A.1.1 step 1.
2. Guess initial values for the threshold $\bar{B}(0)_{pes}$ and derive value function $V_{P,2}^R(B, 0)$.

$$\begin{aligned}
V_{P,2}^R(B, 0) &= \max_{\bar{B}(0)_{pes} < B' \leq \bar{B}(1)} \mathcal{U}(c, g) + \beta \left(pV(B', 1) + (1-p)V^D(0) \right) \\
&\text{s.t.} \quad g + B = \tau A\bar{y} + \beta pB', \\
&\quad c = (1-\tau)A\bar{y}
\end{aligned}$$

3. According to the proof of Proposition 1 in Appendix C, we have $V_{P,2}^R(\bar{B}(0)_{pes}, 0) > V_{P,1}^R(\bar{B}(0)_{pes}, 0)$. Hence, derive $\bar{B}(0)_{pes}$ by solving:

$$V_{P,2}^R(\bar{B}(0)_{pes}, 0) = V^D(0)$$

4. Guess the value function in a pessimistic world $\tilde{V}(B, 0)$ and the threshold B_N .
5. Guess the bond price function $\tilde{q}_{pes}(B', 0)$ in a recession. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B |V(B, 0) - \tilde{V}(B, 0)| < \epsilon$.

$$V(B, 0) = \begin{cases} V_{safe}(B, 0) & \text{if } 0 \leq B \leq B_N \\ V_{P,2}^R(B, 0) & \text{if } B_N < B \leq \bar{B}(0)_{pes} \\ V^D(0) & \text{if } \bar{B}(0)_{pes} < B \end{cases}$$

where

$$\begin{aligned} V_{safe}(B, 0) = \max_{0 \leq B' \leq B_N} & \mathcal{U}(c, g) + \beta \left(pV(B', 1) + (1-p)\tilde{V}(B', 0) \right) \\ \text{s.t.} & \quad g + \kappa B = \tau A\bar{y} + \tilde{q}_{pes}(B', 0)(B' - (1-\delta)B), \\ & \quad c = (1-\tau)A\bar{y} \end{aligned}$$

6. Compute the government utility and policy function given the bond price $q(B', 0) = \beta p$, denoted as $V_P^R(B, 0) = \max\{V_{P,1}^R(B, 0), V_{P,2}^R(B, 0)\}$ and $B'_{pes}(B, 0)$, respectively.

$$\begin{aligned} V_{P,1}^R(B, 0) = \max_{0 \leq B' \leq \bar{B}(0)_{pes}} & \mathcal{U}(c, g) + \beta \left(pV(B', 1) + (1-p)\tilde{V}(B', 0) \right) \\ \text{s.t.} & \quad g + B = \tau A\bar{y} + \beta p B', \\ & \quad c = (1-\tau)A\bar{y} \end{aligned}$$

7. Derive a new value of B_N by solving equation below.

$$B_{N,new} = \sup_B \{B'_{pes}(B, 0) \leq \bar{B}(0)_{pes}\}$$

8. Update the bond price function and compute the error. New values of $q_{pes}(B', 0)$ are

$$q_{pes}(B', 0) = \begin{cases} \beta \left(p + (1-p)(\kappa + (1-\delta)\tilde{q}_{pes}(B'(B', 0), 0)) \right) & \text{if } 0 \leq B' \leq B_N \\ \beta \left(p + (1-p)(\kappa + (1-\delta)\beta p) \right) & \text{if } B_N < B' \leq \bar{B}(0)_{pes} \\ \beta p & \text{if } \bar{B}(0)_{pes} < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

9. If $\max_{B'} |\tilde{q}_{pes}(B', 0) - q_{pes}(B', 0)| > \epsilon$ or/and $|B_N - B_{N,new}| > \epsilon$, then update values: $\tilde{q}_{pes}(B', 0) = q_{pes}(B', 0)$, $B_N = B_{N,new}$, and go back to 5. Else, exit.

B.2 Debt thresholds in the debt-limit version of the model

B.2.1 Optimistic beliefs

1. Derive the debt threshold in normal times. $\bar{B}(1)$ can be characterized by

$$\bar{B}(1) = \frac{\tau\bar{y} - \bar{g}}{1 - \beta}$$

2. Derive the debt threshold in a recession.

$$\bar{B}(0)_{opt} = \max \left\{ \frac{\tau A\bar{y} - \bar{g}}{1 - \beta}, \frac{\tau A\bar{y} - \bar{g} + \beta p \bar{B}(1)}{1 - \beta(1 - p)(1 - \delta)} \right\}$$

3. The methodology to derive the government policy choice and the bond price schedule is similar to the one followed for our baseline, but in two respects. First, in the debt-limit framework, debt thresholds can be computed directly without any iterative procedure. Second, the utility of repaying on finite discretized space in a simulation must be larger than the utility of defaulting. For instance, when sovereign bond space $[0, 200]$ is discretized into finite grid points, the utility of defaulting must be low enough (e.g. $V_d(0) = -99999999$) so that the utility of repaying in discretized space is always larger than utility of defaulting. Computationally, defaulting is never an optimal choice in a simulation—this rules out strategic default.

B.2.2 Pessimistic

1. Repeat A.2.1 step 1.
2. Derive the debt threshold in a recession.

$$\bar{B}(0)_{pes} = \frac{\tau A\bar{y} - \bar{g} + \beta p \bar{B}(1)}{1 - \beta(1 - p)(1 - \delta)}$$

3. Set $V_d(0) = -99999999$ and use the debt thresholds derived in stage 1 and 2 to solve the equilibrium.

C Proofs

C.1 Proof of Proposition 1

Proof. To prove that \mathbb{B}_{pes} is non-empty, we only need to show that there exists some debt level that simultaneously satisfies $V_P^R = \max\{V_{P,1}^R, V_{P,2}^R\} = V_{P,2}^R$ and $V_{P,2}^R \geq V^D(0)$, i.e., the government chooses to issue debt above the threshold $\bar{B}(0)_{pes}$ and repay the existing debt at the default-risky rate.

Posit that, for some $\tilde{B} \leq \bar{B}(0)_{pes}$, the critical expenditure \bar{g} is large enough to exceed $\tau A\bar{y} - \tilde{B} + \beta p\bar{B}(0)_{pes}$. $\bar{B}(0)_{pes}$ is derived by solving the following equation (see $V_{P,2}^R$ in (16)):

$$V_{P,2}^R(\bar{B}(0)_{pes}, 0) = V^D(0) \quad (17)$$

For $B \in [\tilde{B}, \bar{B}(0)_{pes}]$, since $\bar{g} > \tau A\bar{y} - \tilde{B} + \beta p\bar{B}(0)_{pes}$, if the government issued bonds below $\bar{B}(0)_{pes}$ at the risky rate (see $V_{P,1}^R$ in (15)), it would have to cut spending below \bar{g} .

Hence, it must be the case that $V_P^R(B, 0) = V_{P,2}^R(B, 0)$ for all $B \in [\tilde{B}, \bar{B}(0)_{pes}]$. The amount of newly issued bonds B' in this range must be unambiguously larger than $\bar{B}(0)_{pes}$, validating the pessimistic belief.

It follows that, when bonds trade at the default-risky price, a sufficient condition for a non-empty set \mathbb{B}_{pes} is a large enough \bar{g} . Note that the proof above also implies that $\bar{B}(0)_{pes}$ is pinned down by solving the equation $V_{P,2}^R(\bar{B}(0)_{pes}, 0) = V^D(0)$, as referred to the computation algorithm. See Appendix B for details. □

C.2 Proof of Proposition 2

Proof. Define $\mathcal{E}(B) \equiv V_E^R(B, 0) - V^D(0)$. Since $V^D(0)$ is a constant, by the properties of the value of repaying debt, $\mathcal{E}(B)$ is a continuous and monotonically decreasing function. Given that $\lim_{g \rightarrow \bar{g}} \mathcal{U}(c, g) = -\infty$, by the intermediate value theorem the following inequalities imply existence and the uniqueness of a debt threshold $\bar{B}(0)_{EX}$ in the region $(0, \tau A\bar{y} - \bar{g})$:

$$\begin{aligned} \mathcal{E}(0) &> 0 \\ \lim_{B \rightarrow \tau A\bar{y} - \bar{g}} \mathcal{E}(B) &= -\infty \end{aligned}$$

□

C.3 Proof of Proposition 3

Proof. First, we prove that $\bar{B}(0)_{opt}$ is not smaller than $\bar{B}(0)_{pes}$. By proposition 1, when $B = \bar{B}(0)_{pes}$, in a pessimistic world the government always borrows into a default-risky level (above $\bar{B}(0)_{pes}$ but below $\bar{B}(1)$). $\bar{B}(0)_{pes}$ is characterized by solving the equation below (see $V_{P,2}^R$ in (16)):

$$V_{P,2}^R(\bar{B}(0)_{pes}, 0) = V^D(0)$$

In an optimistic world, instead, when the initial debt level is $\bar{B}(0)_{opt}$, the government can choose to either borrow into a risky level (above $\bar{B}(0)_{opt}$ but below $\bar{B}(1)$), or keep the issuance at safe levels (not larger than $\bar{B}(0)_{opt}$).

If the government adopts the risky issuance strategy, $\bar{B}(0)_{opt}$ is characterized by solving:

$$V_{O,2}^R(\bar{B}(0)_{opt}, 0) = V^D(0)$$

(see $V_{O,2}^R$ in (5)). It is easy to verify that $V_{O,2}^R$ and $V_{P,2}^R$ represent the same optimization problem and therefore $\bar{B}(0)_{opt} = \bar{B}(0)_{pes}$.

If the government adopts the safe issuance strategy, it must be the case that this enhances debt sustainability relative to the risky one (see $V_{O,1}^R$ in (4)):

$$V_{O,1}^R(\bar{B}(0)_{pes}, 0) > V_{O,2}^R(\bar{B}(0)_{pes}, 0) = V^D(0)$$

$\bar{B}(0)_{opt}$ in this case is unambiguously larger than $\bar{B}(0)_{pes}$. □

C.4 Proof of Proposition 4

Proof. Rewrite $V_{P,1}^R(B, 0)$ when $B = \bar{B}(0)_{CK}$:

$$\begin{aligned} V_{P,1}^R(\bar{B}(0)_{CK}, 0) = & \max_{0 \leq B' \leq \bar{B}(0)_{pes}} \mathcal{U}(c, g) + \beta \left(pV(B', 1, \rho = P) + (1 - p)V(B', 0, \rho = P) \right) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta pB', \\ & c = (1 - \tau)A\bar{y} \end{aligned}$$

If we set the choice variable B' to zero, the value of $V_{P,1}^R(\bar{B}(0)_{CK}, 0)$ is equal to $V_E^R(\bar{B}(0)_{CK}, 0)$ (see V_E^R in (6)). However, the optimal choice of B' is unambiguously positive in $V_{P,1}^R(\bar{B}(0)_{CK}, 0)$. By strict concavity of \mathcal{U} , this implies that $V_{P,1}^R(\bar{B}(0)_{EX}, 0)$ is larger than $V_E^R(\bar{B}(0)_{EX}, 0)$. Hence, $\bar{B}(0)_{pes} > \bar{B}(0)_{EX}$. □

D Policy functions with short-term bonds

To highlight the role of debt maturity, Figure 12 shows the policy functions conditional on a recession for the case of one-period bonds ($\delta = 1.0$). Comparing this with Figure 4, it is apparent that, as δ converges to unity, $\bar{B}(0)_{pes}$ is much lower, while $\bar{B}(0)_{opt}$ is not affected. The result that $\bar{B}(0)_{opt}$ is not sensitive to debt maturity follows from the fact that, when investors hold an optimistic view on government solvency, they lend the government at risk-free rate: thus there is little scope for maturity to make a difference. Indeed, the left panel in Figure 12 features exactly the same dynamics as the left panel of Figure 4.

Maturity instead makes a difference for the region of debt in which “fast” and “slow crises” are possible. The region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ is much wider with short-term debt. The rollover crises might occur for low levels of debt (above 41% of GDP in normal times). Moreover, a larger region of “fast crises” is not fully compensated by a narrowing of the

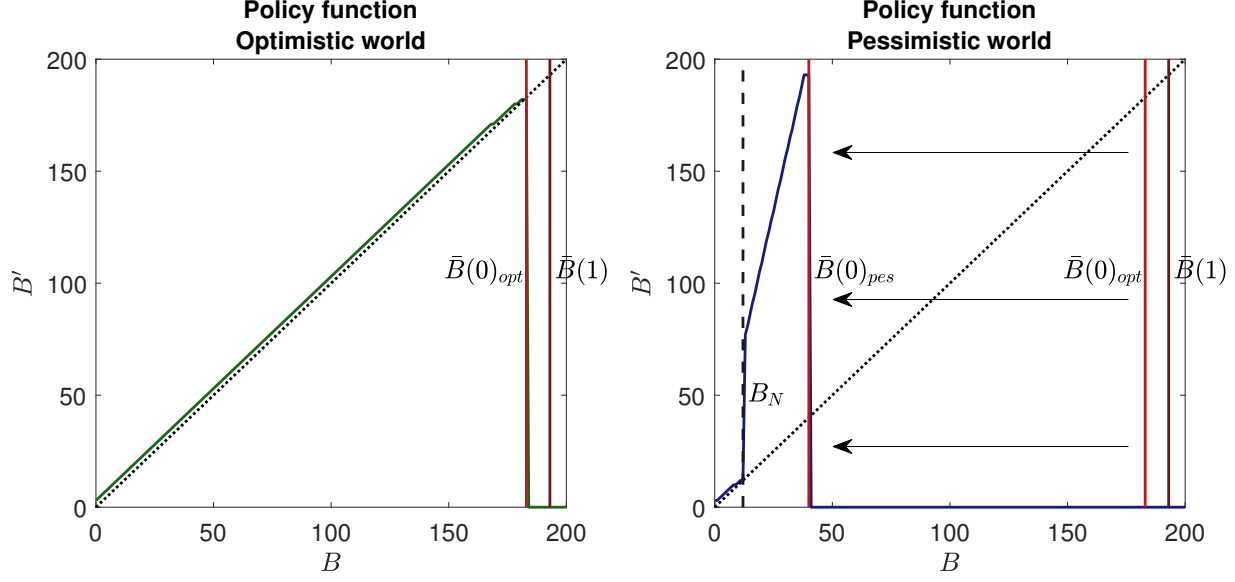


Figure 12: Policy functions for one-period bonds, $\delta = 1.0$

region of “slow crises”, as both B_N and $\bar{B}(0)_{pes}$ shrink when maturity is shorter. B_N falls from 53 to 12! When government bonds are all short-term, a country in a recession might suffer a “slow-moving” crisis even if it has low enough outstanding debt.

E Debt evolution in baseline sunspot equilibria

Figure 13 displays the debt path starting from $B = 75$ in sunspot equilibria. The government accumulates debt over time as long as the sunspot event does not occur. Debt accumulation is slower when debt level is below $\bar{B}(0)_{pes}$, whereas it accelerates after sovereign debt enters “fast” crises zone, above $\bar{B}(0)_{pes}$.

F The ‘Cliff Effect’ in welfare due to self-fulfilling crises

In Figure 14, we show the government value function in a pessimistic world for $\delta = 0.2$. We dub ‘cliff’ the discontinuity in the value function at the debt threshold. A cliff is apparent at B_N . In a sunspot equilibrium, this large loss of utility associated to vulnerability to belief-driven crises motivates the government to deleverage and keep debt at safe levels, below B_N . Observe, however, that there is no cliff around the other, higher threshold — a significant utility incentive for the government to deleverage only exists around B_N .

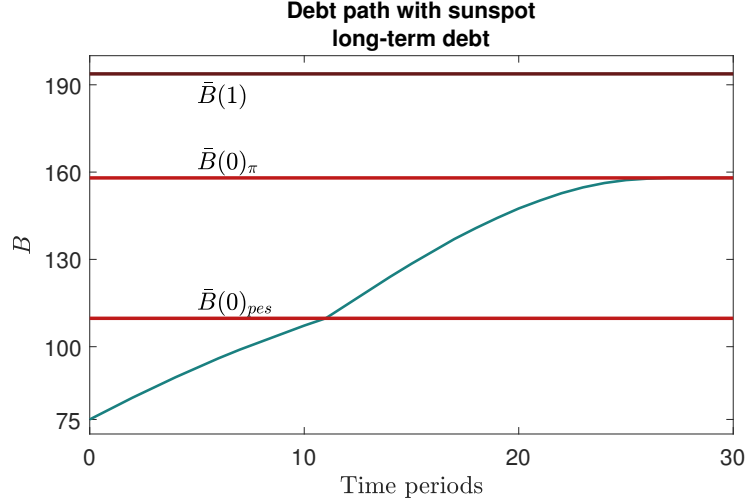


Figure 13: Debt path starting from $B = 75$

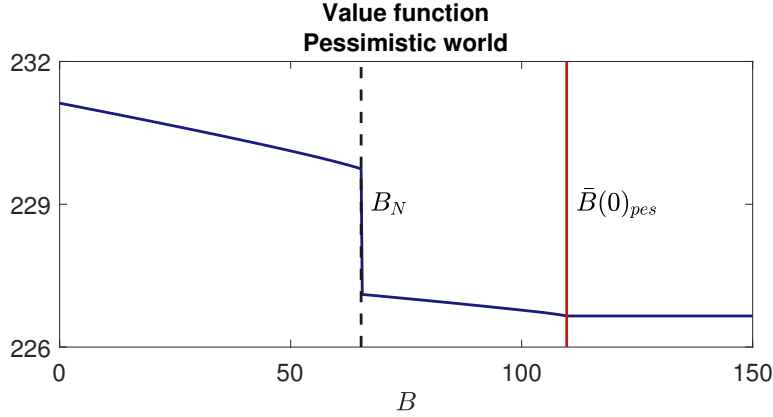


Figure 14: Cliff effect ($\delta = 0.2$)

G Multiplicity in a debt-limit version of our model

In this appendix, we reconsider our main results in a debt-limit framework, which we introduced in the main text by rewriting the sustainability condition as (14). Based on this condition, below we first characterize the debt tolerance thresholds for the case of one-period bonds. Then we derive a debt-limit model as a variant of our baseline, and carry out numerical analysis for generic bond maturity.

G.1 The debt tolerance threshold in the debt-limit framework

In the debt-limit framework, the debt tolerance thresholds are pinned down by the maximum adjustment in primary surpluses the government is willing/able to generate. Yet they are not unique: they may be shifting in response to the regime of investors' expectations. We

show the derivation of thresholds conditional on short debt maturity in the following.

G.1.1 The debt tolerance threshold in normal times $\bar{B}(1)$

In normal times, the government budget constraint is

$$B = \tau \bar{y} - g + q(B', s)B'$$

Since, once the economy recovers, it never falls back into a recession again, there is no reason to borrow or lend for consumption smoothing purposes. The government optimization problem is deterministic when the regime of beliefs is either optimistic or pessimistic ($\rho = O$ or P). If no default has occurred in the past, the government will simply service its existing debt at the risk-free rate, paying $(1 - \beta)B$ to investors each period, to satisfy the no-Ponzi condition.³²

Given τ , the government does not default if and only if

$$B \leq \frac{\tau \bar{y} - \bar{g}}{1 - \beta} = \bar{B}(1)$$

where \bar{g} is the critical expenditure level.

G.1.2 The debt tolerance threshold(s) in a recession $\bar{B}(0)$

In what follows, we derive debt thresholds for optimistic, pessimistic and extreme beliefs respectively.

Pessimistic Beliefs

In a recession, the government budget constraint reflects the decline in tax revenue due to the downturn in activity ($A < 1$):

$$B = \tau A \bar{y} - g + q(B', s)B'$$

In a pessimistic world, investors are only willing to buy bonds at the low risky price. Given the definition of the debt tolerance threshold, the maximum the government can borrow is capped by the stock of debt that the government can service if the economy recovers, that is, $\max\{q(B', s)B'\} = \beta p \bar{B}(1)$. Hence, to rule out immediate default, the current debt level must be low enough to satisfy:

$$B \leq \tau A \bar{y} - \bar{g} + \beta p \bar{B}(1) = \bar{B}(0)_{pes}, \quad (18)$$

³²In normal times, the debt threshold given extreme beliefs in the debt-limit is derived via solving B in the following equation $B = \tau \bar{y} - \bar{g}$. As we do not refer to the threshold in the paper, we leave the result in this footnote.

an expression that gives us the current debt tolerance threshold $\bar{B}(0)_{pes}$. Analogously, we can also derive the threshold B_N below which the government is immune to pessimism:

$$B \leq \tau A\bar{y} - \bar{g} + \beta p \bar{B}(0)_{pes} = B_N.$$

If this condition is satisfied, the government will keep the debt level below $\bar{B}(0)_{pes}$, even if bonds trade at the default-risky price.

Optimistic Beliefs

In an optimistic world, we need to examine two possible issuance strategies for the government. One consists of issuing a lot of debt, at a low, risky price—essentially this is the same strategy as described above, and is therefore associated to the same debt threshold in (18). The other one consists of keeping new issuance in check, so to ensure that debt remains safe. This can be dubbed as a “low-risk low-debt” issuance strategy. By using the same steps above, we can derive the maximum sustainable debt conditional on the safe-debt strategy as:

$$B \leq \frac{\tau A\bar{y} - \bar{g}}{1 - \beta}$$

Thus, $\bar{B}(0)_{opt}$ can be characterized as follows:

$$\bar{B}(0)_{opt} = \max \left\{ \frac{\tau A\bar{y} - \bar{g}}{1 - \beta}, \bar{B}(0)_{pes} \right\}$$

Which strategy gives the government higher revenue in an optimistic world depends on parameters. If all government debt is short-term, we find that $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$, and thus a safe-debt strategy makes the government better off.

Extreme Beliefs

In the eye of investors who hold extreme beliefs, the current debt is sustainable if it satisfies:

$$B \leq \tau A\bar{y} - \bar{g}$$

i.e., it will be repaid even if the government loses market access. Hence, the debt threshold in an extreme world is characterized by

$$\bar{B}(0)_{EX} = \tau A\bar{y} - \bar{g}$$

$\bar{B}(0)_{EX}$ is unambiguously the smallest among the thresholds. The relationship of thresholds is summarized in the following:

$$\bar{B}(0)_{opt} \geq \bar{B}(0)_{pes} > \bar{B}(0)_{EX}$$

G.2 The government welfare function

To study the debt-limit model, we use the following variant of our baseline. The main idea is that the government suffers a utility cost Γ if it cuts spending below \bar{g} . Specifically, we replace (13) with a new objective function:

$$\mathcal{U}(c, g) = \mathbb{1}_{g > \bar{g}} (\log(c) + \gamma \log(g - \bar{g} + \epsilon)) - (1 - \mathbb{1}_{g > \bar{g}}) \times \Gamma,$$

where $\mathbb{1}_{g > \bar{g}}$ is an indicator function equal to 0 if spending falls below the critical value. We assume an arbitrary small positive ϵ to ensure that $\mathcal{U}(c, g)$ is bounded below when $g \rightarrow \bar{g}$. This is the key implication: if defaulting brings spending below the critical level \bar{g} and a utility penalty Γ is cruel enough, the value of repaying will never be below that of defaulting—the government never defaults strategically. Yet, as shown below, crises are still possible, depending on the initial conditions, the persistence of recessions and investors’ expectations.

Using this new framework, we now set $Z = 0.8$, $\tau = 0.35$, $\bar{g} = 30$ such that government spending falls below the critical level \bar{g} upon a default.³³ For the other parameters, we adopt the same values as in the baseline of Table 1. We discuss the case of “static” beliefs in Appendix G.3 and a more general analysis of sunspots in Appendix G.4. We will show that, as further discussed in Section 6, in this debt-limit framework long-term debt tends to rule out “fast” debt crises more easily, but remains ineffective in ruling out “slow-moving” debt crises.

G.3 What difference does a debt-limit framework make?

The main results from our exercise are shown in the two panels of Figure 15, which depicts the policy functions with long-term bonds (left panel) and one-period bonds (right panel). Each panel illustrates both the optimistic and the pessimistic world.

With one-period bonds—the case shown in the right panel of Figure 15—the debt dynamics are very similar to Figure 4, although thresholds are much lower. In an optimistic world, the government accumulates debt over time to smooth consumption till it reaches $\bar{B}(0)_{opt}$. In a pessimistic world, the government issues safe debt at a slow pace in the region between 0 and B_N ; it starts to accumulate risky debt at a fast pace in the region between B_N and $\bar{B}(0)_{pes}$. Fast, rollover crises can nonetheless occur for debt levels between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$.

The debt dynamics shown by the left panel in Figure 15 are quite different. The equilibrium is unique for a low level of debt (in the region between 0 and 74) and for a high level

³³We observe that the initial recessionary state can be quite adverse, i.e., A can be so low that the government cannot finance the critical level of spending \bar{g} without borrowing, i.e., $A\tau\bar{y} < \bar{g}$. We discuss this case in Appendix H.

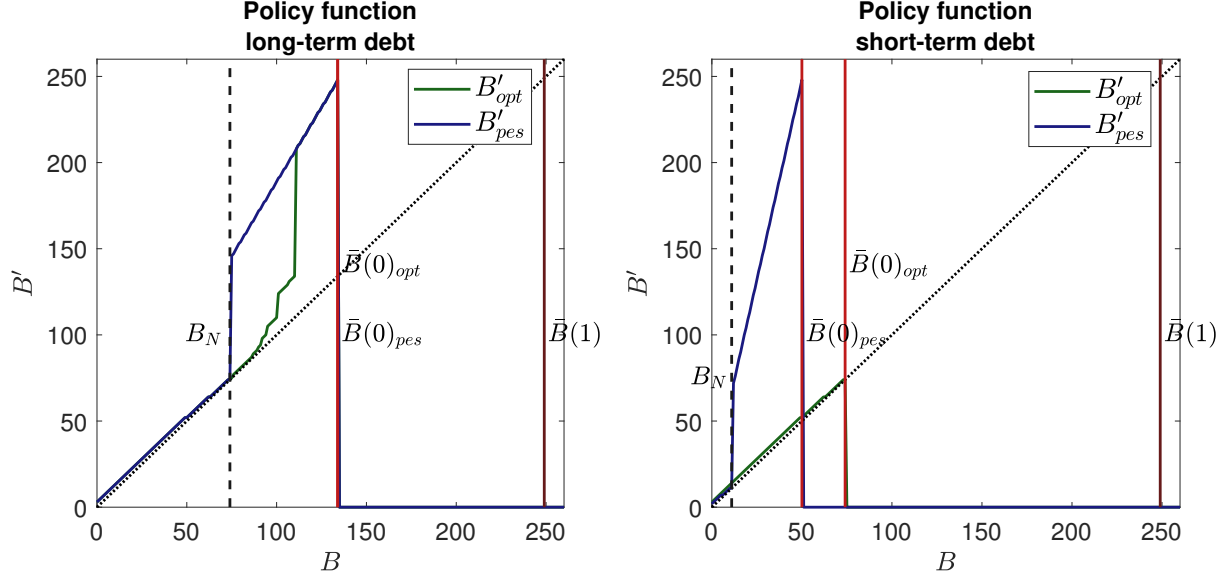


Figure 15: Policy functions where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

of debt (in the region above 110). Multiplicity exists for intermediate levels of debt (in the region between 74 and 110). With long-term debt, $\bar{B}(0)_{pes}$ coincides with $\bar{B}(0)_{opt}$: this rules out the possibility of “fast” debt crises.

This result suggests that debt maturity is much more consequential in the debt-limit than in the baseline model. As further discussed in Section 6, we find that “fast” debt crises are ruled out in the debt-limit version of our model for any δ below 0.57, corresponding to a debt maturity of seven quarters. For longer debt maturities, “none” and “slow” are the only possible outcomes in the debt-limit framework.

G.4 Sunspot equilibria in a debt-limit framework

We study how sunspots can affect government behavior in the debt-limit framework. To keep things simple and for the sake of comparison with [Lorenzoni and Werning \(2019\)](#), in the following we restrict our attention to a scenario in which a switch is possible from optimistic to pessimistic beliefs regime.

In a debt-limit framework, a sunspot equilibrium modifies our previous analysis in two respects. First, when government bonds are long-term, at intermediate levels of debt, there is an acceleration of debt accumulation. Second, when debt is short-term, debt thresholds become sensitive to the probability attributed to the sunspot—they shift at low values of these probabilities.

In Figure 16, we display the bond price function and debt accumulation in the time domain for two different levels of debt in our economy with long-term bonds. Each panel

illustrates both the optimistic equilibrium and the sunspot equilibrium. We omit the policy function from the graph because this is visually very close to an optimistic world in Figure 15.

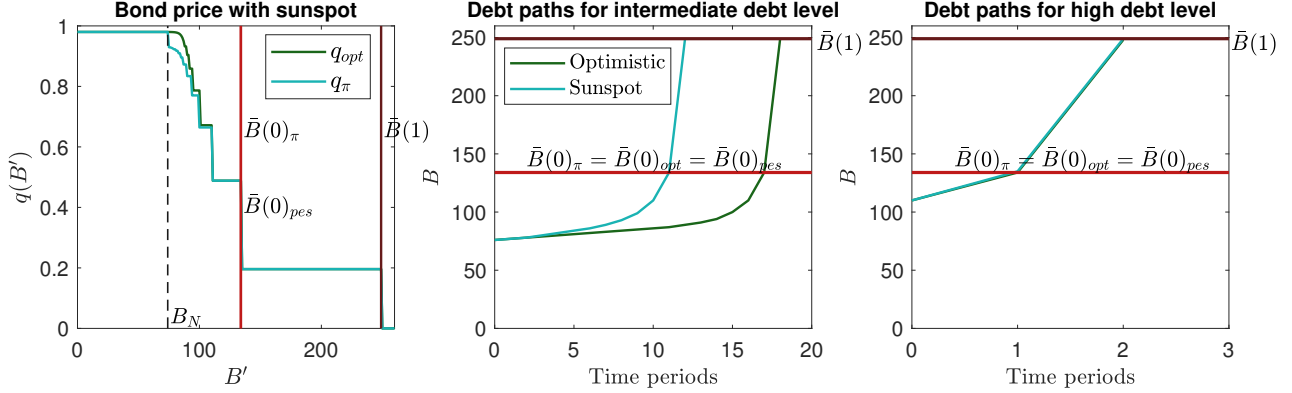


Figure 16: Bond price schedules and debt paths ($\delta = 0.2$)

The center and right panels of Figure 16 clarify the main difference between the optimistic and the sunspot equilibria. In both panels, default may occur with positive probability, but the center panel starts from a moderate debt level ($B = 76$), while the right panel starts from a high debt level ($B = 110$).

The sunspot equilibrium makes a difference only for the case in the center panel of Figure 16: the government accumulates debt faster in a sunspot world. When the economy is exposed to sunspot crises, the government has to pay higher spread at intermediate levels of debt. This accelerates debt accumulation: as debt crises arrive earlier, the spread rises even further, larger than $\pi = 4\%$ for B' close to B_N in the left panel of Figure 16.

When debt level is high enough, pricing in the sunspot equilibrium is less crucial. The right panel of Figure 16 shows that the debt paths are identical in both the sunspot and optimistic equilibria. Intuitively, investors may turn pessimistic at $T = 1$, but the government always chooses risky-debt high-debt issuance strategy at $T = 1$ regardless of investors' beliefs (multiplicity does not exist for high debt levels, see Figure 15). As a result, the sunspot is immaterial for the equilibrium.

The economy with one-period debt features different debt dynamics. Strikingly, $\bar{B}(0)_\pi$ coincides with $\bar{B}(0)_{pes}$: in the sunspot equilibrium, the debt tolerance threshold shrinks towards $\bar{B}(0)_{pes}$ with one-period debt. We find that for any π above 1% (consistent with the policy function), the government always issues default-free debt up to $\bar{B}(0)_{pes}$.

H Deep recessions in a debt-limit framework

In a deep recession, the government is only able to sustain \bar{g} via borrowing ($\bar{g} > \tau A \bar{y}$). A numerical example is shown in Figure 17 where $A = 0.8$. The figure shows the path of optimal debt accumulation over time, contrasting the economy with long-term bonds (left) and one-period bonds (right). The initial debt level is set to 0 in both panels. In either case, the government keeps increasing its debt, even beyond safe levels if a recession persists.

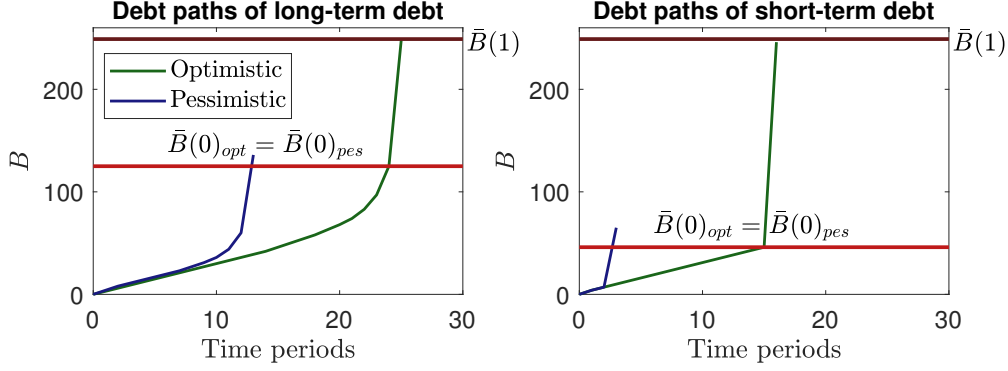


Figure 17: Deep recession $A = 0.8$

Notice that the government accumulates debt faster, and defaults earlier, in a pessimistic world when a recession persists. Comparing these two panels also shows that, when debt is short-term, debt reaches unsustainable levels faster/earlier due to lower $\bar{B}(0)$.

I Ruling out “fast” debt crises in the baseline model

In the sensitivity analysis discussed in the text, we have seen that long debt maturities are effective in eliminating equilibria with fast (rollover) crises mostly in the debt-limit model, not in our baseline. In this appendix we extend our sensitivity analysis and characterize conditions under which the equilibria with fast crises disappear in our baseline. These conditions consist of a combination of three elements: the country is in a very deep recession (A is very low), the probability of recovery is quite high, and debt maturity is sufficiently long—as to mute the pass-through of high interest rates on the total cost of debt servicing. In these conditions, the government has a strong incentive to pursue high-debt risky-debt issuance strategy even when investors’ expectations are optimistic.

In Figure 18 we set $A = 0.8$ and $p = 0.6$: the current recession is exceptionally deep (with a loss of output equal to 20%), but the likelihood of exiting from in a period is larger than 50%. Figure 18 shows policy functions when debt is long-term ($\delta = 0.2$) in the left panel, and when debt is short-term ($\delta = 1.0$) in the right panel.

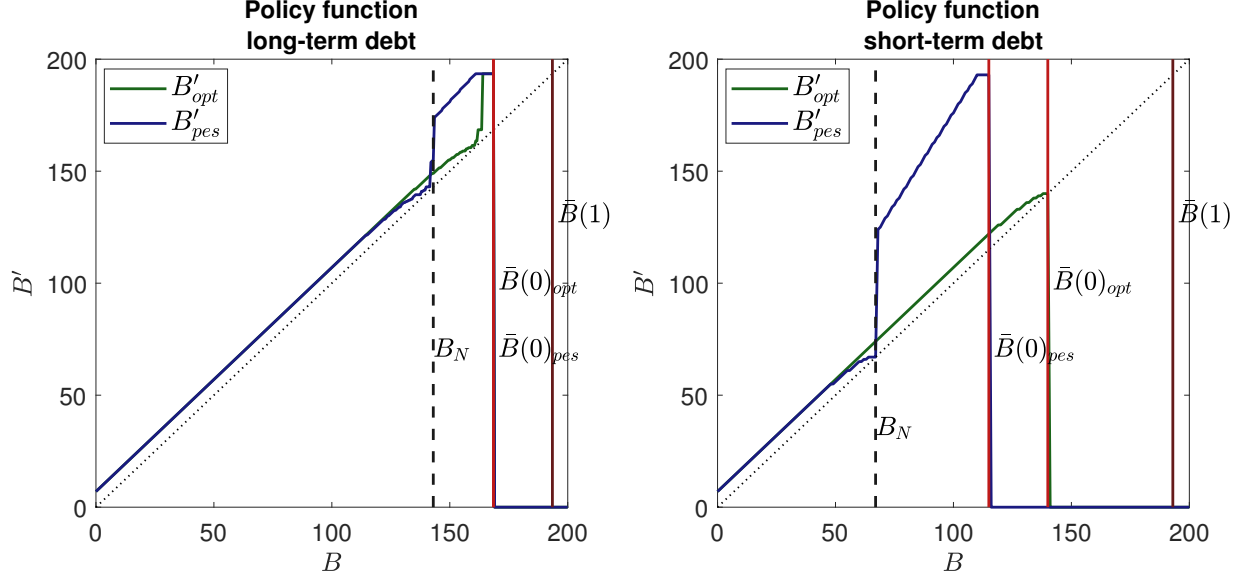


Figure 18: Policy functions in a very severe recession and high probability of recovery, $A = 0.8$ and $p = 0.6$, where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

When government bonds are short-term—the right panel of Figure 18— $\bar{B}(0)_{opt}$ does not coincide with $\bar{B}(0)_{pes}$, and thus fast debt crises are possible. With short debt maturity (high δ), issuing too much debt would quickly raise its interest costs. In an optimistic world, the government prefers stick to a contained, safe-debt issuance strategy.

In contrast, as shown in the left panel of Figure 18, facing a deep recession and a high probability of recovery, the government has an incentive to issue a lot of long-term bonds, which do not need to be fully rolled over in every period. Below $\bar{B}(0)_{opt}$, the government switches to a risky debt issuance and keeps accumulating debt. If, unfortunately, the recession lasts many periods, the government ends up gambling for recovery.

A striking feature of this economy is that, in a pessimistic world, a government with an outstanding debt level below B_N prefers to keep borrowing, even if this means that debt enters the slow-moving crises zone. In the figure, for any debt level below B_N , the government slowly accumulates debt over time, and keeps doing so when debt exceeds B_N . Given a high probability of recovery, the consumption smoothing motive drives the optimal government policy. Once debt is in the region between B_N and $\bar{B}(0)_{pes}$, the government effectively starts gambling on the recovery.